

Chapter 1 : Power series intro (video) | Khan Academy

In this section we will give the definition of the power series as well as the definition of the radius of convergence and interval of convergence for a power series. We will also illustrate how the Ratio Test and Root Test can be used to determine the radius and interval of convergence for a power series.

Due to the nature of the mathematics on this site it is best views in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device should be able to scroll to see them and some of the menu items will be cut off due to the narrow screen width. Power Series and Functions We opened the last section by saying that we were going to start thinking about applications of series and then promptly spent the section talking about convergence again. With this section we will start talking about how to represent functions with power series. The natural question of why we might want to do this will be answered in a couple of sections once we actually learn how to do this. This idea of convergence is important here. In this way we will hopefully become familiar with some of the kinds of manipulations that we will sometimes need to do when working with power series. Example 1 Find a power series representation for the following function and determine its interval of convergence. This is actually easier than it might look. All we need to do now is a little simplification. More often than not the new interval of convergence will be different from the original interval of convergence. Example 2 Find a power series representation for the following function and determine its interval of convergence. The difference is the numerator and at first glance that looks to be an important difference. This is an important idea to remember as it can often greatly simplify our work. Example 3 Find a power series representation for the following function and determine its interval of convergence. So, hopefully we now have an idea on how to find the power series representation for some functions. We now need to look at some further manipulation of power series that we will need to do on occasion. We need to discuss differentiation and integration of power series. With infinite sums there are some subtleties involved that we need to be careful with but are somewhat beyond the scope of this course. We can now find formulas for higher order derivatives as well now. Example 4 Find a power series representation for the following function and determine its interval of convergence. Example 5 Find a power series representation for the following function and determine its interval of convergence.

Chapter 2 : Calculus II - Power Series and Functions

In mathematics, the radius of convergence of a power series is the radius of the largest disk in which the series converges. It is either a non-negative real number or ∞ $\{\displaystyle \infty\}$.

If we refuse to truncate a Taylor polynomial, and instead allow it to be a series an infinite sum we call it a power series. Which of the following are power series functions? Every polynomial is a power series. Here are four basic power series centered at zero that every mathematician knows. Next to each of the series, we list an interval which will correspond to the domain of the series. Here are some examples. We can easily see that e^x , $\cos(x)$, and $\sin(x)$. Since every power of x in the power series for sine is odd, we can see that sine is an odd function. Likewise, since every power of x in the power series for cosine is even, we can see cosine is an even function.

Convergence of power series You may have noticed a small caveat above. This restriction is required because if our formula is true, then for any number x , provided that $|x| < R$, we have and the expression on the right-hand side of the equation above is a geometric series! If we look at a graph of e^x along with a graph of $\sin(x)$ we see True or False: Convergence of Power Series Consider the power series Exactly one of the following is true: A power series always converges when $|x| < R$. In this way, we can use all of our previous tools for working with series. We can also let x be a variable, and consider our power series as a function. Because power series can define functions, we no longer exclusively talk about convergence at a point, instead we talk about the radius and interval of convergence. If a power series converges absolutely for all x , then its radius of convergence is said to be ∞ and the interval of convergence is $(-\infty, \infty)$. If a power series converges absolutely for all x in $(-R, R)$ and diverges for all $|x| > R$, then its radius of convergence is said to be R and the interval of convergence is one of the following: If a power series converges only at one value $x = c$, then its radius of convergence is said to be 0 and the series does not have an interval of convergence. In the previous definition, the interval of convergence depends on the series. We must separately consider the behavior of a power series at the endpoints of its interval of convergence. In other words, we plug in values for $x = -R$ and $x = R$, and consider the series as a series of numbers! Suppose you know that $\sum a_n x^n$ converges when $x = -R$ and diverges when $x = R$. Must the series converge at $x = c$? How do we check for radius of convergence? Two old friends can come to the rescue: Consider the power series: Determine the radius and interval of convergence. For this power series we will use the ratio test. Since the ratio test requires positive terms, we must look at the absolute values of the terms in the series. Now, for any fixed value of x , we have that $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1} x^{n+1}}{a_n x^n} \right| = L$ since we recall that L is a constant in this limit and its value does not affect the value of the limit. Hence, the radius of convergence for $\sum a_n x^n$ is $R = \frac{1}{L}$, and the interval of convergence is $(-R, R)$. While the ratio and root test are good for determining the radius of convergence of a power series, they are useless for determining convergence at the end-points of the interval. Again, we must first use the absolute value of the terms in the series: In other words, the series converges absolutely when $|x| < R$. However, and so adding to all sides of the inequality, we need such that $|x| < R$. Since our power series is centered at c , the radius of convergence is R . However, the root test and ratio test is inconclusive at the end points $x = -R$ and $x = R$. For this, we need to investigate separately the following two series, found by plugging in $x = -R$ and $x = R$. For the first, where $x = -R$, note that This is the alternating harmonic series, which we know converges. So our power series converges at $x = -R$. For the second, where $x = R$, note that This is the harmonic series, which we know diverges. So our power series diverges at $x = R$. Hence the interval of convergence for $\sum \frac{1}{n} x^n$ must include everything between -1 and 1 , as well as -1 , but does not include 1 . In other words, the interval of convergence is $[-1, 1)$. This limit diverges unless $x = c$, the center of the power series. The the radius of convergence is 0 , and there is no interval of convergence, since the series only converges at a single point. New power series from old With the basic power series above, we can produce new power series via algebraic manipulation. Algebra of Power Series Let $\sum a_n x^n$ converge absolutely for $|x| < R$, and let $f(x)$ be a continuous function.

Chapter 3 : Calculus II - Power Series

Power series is a sum of terms of the general form $\sum_{k=0}^{\infty} a_k(x-a)^k$. Whether the series converges or diverges, and the value it converges to, depend on the chosen x -value, which makes power series a function.

Due to the nature of the mathematics on this site it is best views in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device should be able to scroll to see them and some of the menu items will be cut off due to the narrow screen width. In this section we are going to start talking about power series. This will not change how things work however. Everything that we know about series still holds. Before we get too far into power series there is some terminology that we need to get out of the way. This number is called the radius of convergence for the series. What happens at these points will not change the radius of convergence. These two concepts are fairly closely tied together. Note that we had to strip out the first term since it was the only non-zero term in the series. Example 1 Determine the radius of convergence and interval of convergence for the following power series. From this we can get the radius of convergence and most of the interval of convergence with the possible exception of the endpoints. With all that said, the best tests to use here are almost always the ratio or root test. Notice that we now have the radius of convergence for this power series. These are exactly the conditions required for the radius of convergence. All we need to do is determine if the power series will converge or diverge at the endpoints of this interval. The way to determine convergence at these points is to simply plug them into the original power series and see if the series converges or diverges using any test necessary. So, in this case the power series will not converge for either endpoint. The power series could converge at either both of the endpoints or only one of the endpoints. Example 2 Determine the radius of convergence and interval of convergence for the following power series. In other words, we need to factor a 4 out of the absolute value bars in order to get the correct radius of convergence. So, the power series converges for one of the endpoints, but not the other. Example 3 Determine the radius of convergence and interval of convergence for the following power series. If you think about it we actually already knew that however. Example 4 Determine the radius of convergence and interval of convergence for the following power series. Example 5 Determine the radius of convergence and interval of convergence for the following power series. We will usually skip that part.

Chapter 4 : Radius of convergence - Wikipedia

RADIUS OF CONVERGENCE Let be a power series. Then there exists a radius $R \geq 0$ for which (a) The series converges for, and $k \leq R$.

Chapter 5 : The Radius of Convergence etc.

Power Series Calculator Find convergence interval of power series step-by-step.

Chapter 6 : Power series - Ximera

In our example, the center of the power series is 0, the interval of convergence is the interval from -1 to 1 (note the vagueness about the end points of the interval), its length is 2, so the radius of convergence equals 1.

Chapter 7 : Power Series Calculator - Symbolab

Radius of Convergence. A power series will converge only for certain values racedaydvl.com instance, converges racedaydvl.com general, there is always an interval in which a power series converges, and the number is called the radius of convergence (while the interval itself is called the interval of convergence).

Chapter 8 : Power series - Wikipedia

*The calculator will find the radius and interval of convergence of the given power series. Show Instructions In general, you can skip the multiplication sign, so `5x` is equivalent to `5*x`.*

Chapter 9 : Intervals of Convergence of Power Series

If a power series with radius of convergence r is given, one can consider analytic continuations of the series, i.e. analytic functions f which are defined on larger sets than $\{x: |x - c| < r\}$ power series on this set.