DOWNLOAD PDF MATHEMATICS, A DEVELOPMENTAL APPROACH

Chapter 1 : Hawkes Learning | Products | Developmental Mathematics

Developmental Mathematics: A New Approach By William W. Adams. We begin with a familiar story. A student, we call him Tom, arrives at the University, happy to begin his college adventure.

Psychosexual development Sigmund Freud believed that we all had a conscious, preconscious, and unconscious level. In the conscious, we are aware of our mental process. The preconscious involves information that, though not currently in our thoughts, can be brought into consciousness. Lastly, the unconscious includes mental processes we are unaware of. He believed there is tension between the conscious and unconscious because the conscious tries to hold back what the unconscious tries to express. To explain this he developed three personality structures: The id, the most primitive of the three, functions according to the pleasure principle: The first is the oral stage, which occurs from birth to 12 months of age. The second is the anal stage, from one to three years of age. During the anal stage, the child defecates from the anus and is often fascinated with their defecation. During the phallic stage, the child is aware of their sexual organs. The fourth is the latency stage, which occurs from age five until puberty. Stage five is the genital stage , which takes place from puberty until adulthood. During the genital stage, puberty starts happening. He used Socratic questioning to get children to reflect on what they were doing, and he tried to get them to see contradictions in their explanations. Piaget believed that intellectual development takes place through a series of stages, which he described in his theory on cognitive development. Each stage consists of steps the child must master before moving to the next step. He believed that these stages are not separate from one another, but rather that each stage builds on the previous one in a continuous learning process. He proposed four stages: Though he did not believe these stages occurred at any given age, many studies have determined when these cognitive abilities should take place. The pre-conventional moral reasoning is typical of children and is characterized by reasoning that is based on rewards and punishments associated with different courses of action. Conventional moral reason occurs during late childhood and early adolescence and is characterized by reasoning based on rules and conventions of society. Mistrust" takes place in infancy. The second stage is "Autonomy vs. Shame and Doubt" with the best virtue being will. This takes place in early childhood where the child learns to become more independent by discovering what they are capable of where if the child is overly controlled, they believe to feel inadequate on surviving by themselves, which can lead to low self-esteem and doubt. The third stage is "Initiative vs. The basic virtue that would be gained is the purpose and takes place in the play age. This is the stage where the child will be curious and have many interactions with other kids. They will ask many questions as their curiosity grows. If too much guilt is present, the child may have a slower and harder time interacting with other children. The fourth stage is "Industry competence vs. The basic virtue for this stage is competency which happens at the school age. This stage is when the child will try to win the approval of others and fit in and understand the value of their accomplishments. The fifth stage is "Identity vs. The basic virtue gained is fidelity which takes place in adolescence. The sixth stage is "Intimacy vs. Isolation", which happens in young adults and the virtue gained is love. In not doing so, it could lead to isolation. The seventh stage is "Generativity vs. This happens in adulthood and the virtue gained would be care. We become stable and start to give back by raising a family and becoming involved in the community. The eighth stage is "Ego Integrity vs. This happens during maturity and wisdom is gained. When one grows old and they contemplate and look back and see the success or failure of their life. This is also the stage where one can also have closure and accept death without fearing anything. The Model of Hierarchical Complexity MHC is not based on the assessment of domain-specific information, It divides the Order of Hierarchical Complexity of tasks to be addressed from the Stage performance on those tasks. The order of hierarchical complexity of tasks predicts how difficult the performance is with an R ranging from 0. In the MHC, there are three main axioms for an order to meet in order for the higher order task to coordinate the next lower order task. Axioms are rules that are followed to determine how the MHC orders actions to form a hierarchy. Ecological systems theory[edit] Main article: The four systems are microsystem, mesosystem, exosystem, and macrosystem. Each system contains roles, norms and rules that can powerfully shape development. The microsystem is the direct environment in our lives such as our home and school. Mesosystem is how relationships connect to the microsystem. Exosystem is a larger social system where the child plays no role. Macrosystem refers to the cultural values, customs and laws of society. The mesosystem is the combination of two microsystems and how they influence each other example: The exosystem is the interaction among two or more settings that are indirectly linked example: The macrosystem is broader taking into account social economic status, culture, beliefs, customs and morals example: Lastly, the chronosystem refers to the chronological nature of life events and how they interact and change the individual and their circumstances through transition example: As a result of this conceptualization of development, these environmentsâ€”from the family to economic and political structuresâ€”have come to be viewed as part of the life course from childhood through to adulthood. This adult role is often referred to as the skilled "master," whereas the child is considered the learning apprentice through an educational process often termed " cognitive apprenticeship " Martin Hill stated that "The world of reality does not apply to the mind of a child. Constructivism psychological school Constructivism is a paradigm in psychology that characterizes learning as a process of actively constructing knowledge. Individuals create meaning for themselves or make sense of new information by selecting, organizing, and integrating information with other knowledge, often in the context of social interactions. Constructivism can occur in two ways: Individual constructivism is when a person constructs knowledge through cognitive processes of their own experiences rather than by memorizing facts provided by others. Social constructivism is when individuals construct knowledge through an interaction between the knowledge they bring to a situation and social or cultural exchanges within that content. Piaget proposed that learning should be whole by helping students understand that meaning is constructed. Evolutionary developmental psychology Evolutionary developmental psychology is a research paradigm that applies the basic principles of Darwinian evolution , particularly natural selection , to understand the development of human behavior and cognition. It involves the study of both the genetic and environmental mechanisms that underlie the development of social and cognitive competencies, as well as the epigenetic gene-environment interactions processes that adapt these competencies to local conditions. Attachment theory Attachment theory, originally developed by John Bowlby , focuses on the importance of open, intimate, emotionally meaningful relationships. A child who is threatened or stressed will move toward caregivers who create a sense of physical, emotional and psychological safety for the individual. Attachment feeds on body contact and familiarity. Later Mary Ainsworth developed the Strange Situation protocol and the concept of the secure base. Theorists have proposed four types of attachment styles: It is characterized by trust. Anxious-avoidant is an insecure attachment between an infant and a caregiver. Anxious-resistant is an insecure attachment between the infant and the caregiver characterized by distress from the infant when separated and anger when reunited. Some babies are raised without the stimulation and attention of a regular caregiver or locked away under conditions of abuse or extreme neglect. The possible short-term effects of this deprivation are anger, despair, detachment, and temporary delay in intellectual development. Long-term effects include increased aggression, clinging behavior, detachment, psychosomatic disorders, and an increased risk of depression as an adult. Attachment is established in early childhood and attachment continues into adulthood. An example of secure attachment continuing in adulthood would be when the person feels confident and is able to meet their own needs. An example of anxious attachment during adulthood is when the adult chooses a partner with anxious-avoidant attachment. Please help improve this article by adding citations to reliable sources. Unsourced material may be challenged and removed. April Learn how and when to remove this template message Nature vs nurture[edit] A significant issue in developmental psychology is the relationship between innateness and environmental influence in regard to any particular aspect of development. This is often referred to as " nature and nurture " or nativism versus empiricism. An empiricist perspective would argue that those processes are acquired in interaction with the environment. Today developmental psychologists rarely take such polarised positions with regard to most aspects of development; rather they investigate, among many other things, the relationship between innate and environmental influences. One of the ways this relationship has been explored in recent years is through the emerging field of evolutionary developmental psychology. One area where this innateness debate has been prominently portrayed is in research on language acquisition.

A major question in this area is whether or not certain properties of human language are specified genetically or can be acquired through learning. The empiricist position on the issue of language acquisition suggests that the language input provides the necessary information required for learning the structure of language and that infants acquire language through a process of statistical learning. From this perspective, language can be acquired via general learning methods that also apply to other aspects of development, such as perceptual learning. The nativist position argues that the input from language is too impoverished for infants and children to acquire the structure of language. Linguist Noam Chomsky asserts that, evidenced by the lack of sufficient information in the language input, there is a universal grammar that applies to all human languages and is pre-specified. This has led to the idea that there is a special cognitive module suited for learning language, often called the language acquisition device.

Chapter 2 : The Role of Manipulative Materials in the Learning of Mathematical Concepts

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What does the workbook offer my students? Conceptual exercises to extend understanding Application exercises with real life contexts Study Skills to improve student success Group or individual chapter discovery projects Quick Tips to provide helpful problem-solving hints Lesson Links -- A connection of current topics to previously learned topics or future topics Math Work -- A connection of the skills learned to career contexts Foundations Skill Check -- A chapter self-test for knowledge of previously learned skills needed for the next chapter Why the Quantitative Reasoning Approach? As an instructor, you are probably asking yourself if this workbook is the right choice for your developmental math students. A Quantitative Reasoning Approach workbook was developed for instructors like yourself, who are looking for a better way to teach developmental students the vital foundational topics in mathematics that students need to progress to college-level mathematics courses such as Math for Liberal Arts, College Algebra, Statistics, and Quantitative Reasoning. Instead of focusing on rote memorization of procedures, which are soon forgotten once the class is over, this workbook takes a different approach than most developmental texts. Our workbook focuses more on the thorough understanding of concepts and the use of reasoning skills to solve problems. By focusing on concepts, we aim to build understanding. By integrating technology, we aim to actively engage students in their own learning. Our vision while creating this workbook was to more efficiently and effectively prepare students for the mathematics courses they will take in college and to hopefully change their perspective and way of thinking about mathematics as a whole. In addition, we hope that students who complete this workbook will no longer think of mathematics as something they must endure in order to reach their educational goals, but instead as a subject they need to master in order to make their lives and careers more productive and rewarding. How do I use this workbook? For students who need an intensive two-course sequence in foundational mathematics. For students at institutions desiring an alternate pathway to achieve mathematical literacy or reasoning skills as a precursor to a curriculum-level course in Quantitative Reasoning or Statistics. Material from Chapters can be used for review purposes. The combination of the workbook and software is flexible enough to accommodate a variety of instructor teaching styles and institutional course offerings: Flipped Classroom Students read through the Learn portion on their own. The instructor floats around the room offering guidance as needed. Additional practice in the software can be done outside of class. Traditional Lecture or Interactive Classroom Students should preview the Learn portion in the software before the material is presented in class to provide a foundation for the lecture. The amount of time spent lecturing versus in-class work time is up to the individual instructor. Emporium Model Students should read through the Learn portion of the software on their own. Students can use class time to work particular sections of the workbook or complete the Practice mode of the software while they have someone to assist them. If an instructor requires the student to complete Certify mode, we recommend that this be completed in a supervised lab setting or testing center. The instructor answers their questions and provides further instruction if needed. An instructor who chooses to use Certify mode can allow students to complete it at home or require them to complete it in a supervised lab setting or at a testing center on campus. Additional materials or resources needed by students can be posted online. Entirely Online Students would complete the Learn and Practice modes online. Particular sections or exercises from the workbook can be used for additional assignments or discussion forums. Information in the study skills pages can also be used for discussion forums and written assignments. Students could be asked to write a paper discussing their chosen career path and how mathematics might be used in their chosen career. It is left up to the instructor whether they want to use Certify mode and how they would use it in the online environment. The exercises and projects in the Foundations of Mathematics workbook can be used to ensure conceptual understanding, assist in developing problem-solving skills, provide real-world applications of the concepts studied, and provide the necessary study skills for student success.

Chapter 3 : Developmental psychology - Wikipedia

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How Young Children Approach Math Mathematical experiences for very young children should build largely upon their play and the natural relationships between learning and life in their daily activities, interests, and questions. Passing by, her teacher inquires, "Where are the others? Exploring the Math in Play Children become intensely engaged in play. Pursuing their own purposes, they tend to tackle problems that are challenging enough to be engrossing yet not totally beyond their capacities. Sticking with a problem â€" puzzling over it and approaching it in various ways â€" can lead to powerful learning, in addition, when several children grapple with the same problem, they often come up with different approaches, discuss various strategies, and learn from one another. These aspects of play can promote thinking and learning in mathematics as well as in other areas. Young children explore patterns and shapes, compare sizes, and count things. But how often do they do that? When children were studied during free play, six categories of mathematics content emerged. One girl, Anna, took out all the plastic bugs from the container and sorted them by type of bug and then by color. Exploring magnitude describing and comparing the size of objects. Enumerating saying number words, counting, instantly recognizing a number of objects, or reading or writing numbers. Three girls drew pictures of their families and discussed how many brothers and sisters they had and how old their siblings were. Investigating dynamics putting things together, taking them apart, or exploring motions such as flipping. Several girls flattened a ball of clay into a disk, cut it, and made "pizza. Studying pattern and shape identifying or creating patterns or shapes, or exploring geometric properties. Jennie made a bead necklace, creating a yellow-red color pattern. Exploring spatial relations describing or drawing a location or direction. When Teresa put a dollhouse couch beside a window, Katie moved it to the center of the living room, saying, "The couch should be in front of the TV. We can see that free play offers a rich foundation on which to build interesting mathematics. These everyday experiences form the foundation for later mathematics. Later, children elaborate on these ideas. We call this process "mathematization. Play does not guarantee mathematical development, but it offers rich possibilities. Significant benefits are more likely when teachers follow up by engaging children in reflecting on and representing the mathematical ideas that have emerged in their play. Towers of Learning The benefits of block building are deep and broad. Children increase their math, science, and general reasoning abilities when building with blocks. Consider how block building develops. Infants show little interest in stacking. Stacking begins at 1 year, when infants show their understanding of the spatial relationship "on. At 2 years, children place each successive block on or next to the one previously placed. They appear to recognize that blocks do not fall when placed this way. Children begin to reflect and anticipate. At 3 to 4 years of age, children regularly build vertical and horizontal components within a building. When asked to build a tall tower, they use long blocks vertically, because, in addition to aiming to make a stable tower, their goal is to make a stable tall tower, first using only one block in this fashion, then several. At 4 years, they can use multiple spatial relations, extending their buildings in multiple directions and with multiple points of contact among the blocks, showing flexibility in how they build and integrate parts of the structure. Preschoolers employ, at least at the intuitive level, more sophisticated geometric concepts than most children experience throughout elementary school through block play. For example, one preschooler, Jose, puts a double unit block on the rug, two unit blocks on the double unit block, and a triangle unit on the middle, building a symmetrical structure. Consider a preschooler who is making the bottom floor of a block building. He lays two long blocks down, going in the same direction. Then he tries to bridge across the two ends with a short block. However, before he tries the short block again, he carefully adjusts the other end of the long block. He tries the short block. He quickly places many short blocks, creating the floor of his building. We learn a lot from this episode and others like it. Just as this little boy did, many children intuitively use the concepts of parallel and perpendicular. The boy even seems to understand-in his actions-that parallel lines are always the same distance apart! We have observed other children adjusting two cylinders so that the distance between them just equals the length of a long block. They estimate how many more blocks they need to finish a surface. They estimate that eight blocks were needed if each of four sizes of a square are covered with two blocks. We know many math teachers who would be thrilled if their students showed similar insight into geometry, measurement, and number! Rhythm and Patterns Preschoolers also engage in rhythmic and musical patterns. They can add more complicated, deliberate patterns, such as "clap, clap, slap; clap, clap slap" to their repertoires. They can talk about these patterns, representing the pattern with words. Kindergartners enjoy making up new motions to fit the same pattern, so clap, clap slap is transformed into jump, jump, fall down; jump, jump, fall down, and soon, is symbolized as an AABAAB pattern. Kindergartners can also describe such patterns with numbers "two of something, then one of something else". These are actually the first clear links among patterns, number, and algebra. Children who have had these rhythmical experiences will intentionally recreate and discuss patterns in their own artwork. One 4-year-old loved knowing the rainbow colors ROY G BFV, for red, orange, yellow, green, blue, indigo, violet and painted rainbows, flowers, and designs that repeated this sequence several times. A researcher tells of visiting two classrooms in the same day, observing water play in both. Children were pouring in each room, but in one they were also excitedly filling different containers with the same cup, counting how many cupfuls they could "fit" in each container. The only difference between the two classes was that in the latter the teacher had passed by and casually asked, "I wonder which of these holds the most cupfuls of water? Materials such as sand and play dough offer many rich opportunities for mathematical thinking and reasoning. Teachers can provide suggestive materials cookie cutters , engage in parallel play with children, and raise comments or questions regarding shapes and numbers of things. For example, they might make multiple copies of the same shape in play dough with the cookie cutters, or transform sand or play dough into different objects. One teacher told two boys she was going to "hide" the ball of play dough, covering it with a flat piece and pressing down. The boys said the ball was still there, but when she lifted the piece, the ball was "gone. Children at first are unable to combine shapes. They gradually learn to see both individual pieces and a "whole," and learn that parts can make a whole and still be parts. By about 4 years old, most can solve puzzles by trial and error and make pictures with shapes placed next to one another. With experience, they gradually learn to combine shapes to make larger shapes. They become increasingly intentional, building mental images of the shapes and their attributes, such as side length and angles. Building Concepts With Computers Picture making with shapes can be done with building blocks as well as computer shapes. Computer versions have the advantage of offering immediate feedback. For example, shapes can be transparent so children can see the puzzle beneath them. In addition, children often talk more and explain more of what they are doing on computers than when using other materials. At higher levels, computers allow children to break apart and put together shapes in ways not possible with physical blocks. Computers can help facilitate play in other ways, too. The addition of a computer center does not disrupt ongoing play but eases positive social interaction and cooperation. Research shows that computer activity is more effective in stimulating vocalization than play with toys, and also stimulates higher levels of social play. Also, cooperative play at the computer is similar to the amount of cooperative play in the block center. Cooperation in a computer center can provide a context for initiating and sustaining interaction that can be transferred to play in other areas as well, particularly for boys. Dramatic Mathematics Dramatic play can be naturally mathematical with the right setting. In one classroom, Gabi was the shopkeeper. Tamika handed her a five card 5 dots and the numeral "5" as her order. Gabi counted out five toy dinosaurs. Teacher just entering the area: How many did you buy? How do you know? The play allowed her to develop her knowledge. She handed Gabi a 2 and a 5 card. You could give Janelle 2 of one kind and 5 of another. As Gabi counted out the two separate piles and put them in a basket, Janelle counted out dollars. Studies also show that if children play with objects before they are asked to solve problems with them, they are more successful and more creative. For example, one study with three groups of 3- to 5-year-olds asked them to retrieve an object with short sticks and connectors. One group was allowed to play with the sticks and connecting devices, one group was taught how the sticks could be connected, and one group was asked to tackle the task without prior play or learning. The first two groups performed similarly and achieved better results than the third group. Often, the group that simply played with the sticks and connectors first solved the problem more quickly than the group that was taught how to use them. Mathematical Play This brings us to the final fascinating and usually overlooked type of play:

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Chapter 4 : Hawkes Learning | Products | Foundations of Mathematics: A Quantitative Reasoning Approac

Mathematics for the Lincoln, Nebraska, A DEVELOPMENTAL APPROACH TO PREPARING STUDENTS FOR STANDARDIZED OR STATE TESTS. DEVELOP STUDENTS' COMFORT AND.

We now turn our attention to what it takes to develop proficiency in teaching mathematics. Proficiency in teaching is related to effectiveness: Proficiency also entails versatility: Teaching in the ways portrayed in chapter 9 is a complex practice that draws on a broad range of resources. Despite the common myth that teaching is little more than common sense or that some people are just born teachers, effective teaching practice can be learned. In this chapter, we consider what teachers need to learn and how they can learn it. First, what does it take to be proficient at mathematics teaching? If their students are to develop mathematical proficiency, teachers must have a clear vision of the goals of instruction and what proficiency means for the specific mathematical content they are teaching. They need to know the mathematics they teach as well as the horizons of that mathematicsâ€"where it can lead and where their students are headed with it. They need to be able to use their knowledge flexibly in practice to appraise and adapt instructional materials, to represent the content in honest and accessible ways, to plan and conduct instruction, and to assess what students are learning. Helping Children Learn Mathematics. The National Academies Press. If you can interweave the two things together nicely, you will succeedâ€¦. Believe me, it seems to be simple when I talk about it, but when you really do it, it is very complicated, subtle, and takes a lot of time. It is easy to be an elementary school teacher, but it is difficult to be a good elementary school teacher. Used by permission from Lawrence Erlbaum Associates. Teaching requires the ability to see the mathematical possibilities in a task, sizing it up and adapting it for a specific group of students. In short, teachers need to muster and deploy a wide range of resources to support the acquisition of mathematical proficiency. In the next two sections, we first discuss the knowledge base needed for teaching mathematics and then offer a framework for looking at proficient teaching of mathematics. In the last two sections, we discuss four programs for developing proficient teaching and then consider how teachers might develop communities of practice. The Knowledge Base for Teaching Mathematics Three kinds of knowledge are crucial for teaching school mathematics: Page Share Cite Suggested Citation: In our use of the term, knowledge of mathematics includes consideration of the goals of mathematics instruction and provides a basis for discriminating and prioritizing those goals. Knowing mathematics for teaching also entails more than knowing mathematics for oneself. Teachers certainly need to be able to understand concepts correctly and perform procedures accurately, but they also must be able to understand the conceptual foundations of that knowledge. In the course of their work as teachers, they must understand mathematics in ways that allow them to explain and unpack ideas in ways not needed in ordinary adult life. Knowledge of students and how they learn mathematics includes general knowledge of how various mathematical ideas develop in children over time as well as specific knowledge of how to determine where in a developmental trajectory a child might be. Knowledge of instructional practice includes knowledge of curriculum, knowledge of tasks and tools for teaching important mathematical ideas, knowledge of how to design and manage classroom discourse, and knowledge of classroom norms that support the development of mathematical proficiency. Teaching entails more than knowledge, however. Teachers need to do as well as to know. For example, knowledge of what makes a good instructional task is one thing; being able to use a task effectively in class with a group of sixth graders is another. Understanding norms that support productive classroom activity is different from being able to develop and use such norms with a diverse class. Knowledge of Mathematics Because knowledge of the content to be taught is the cornerstone of teaching for proficiency, we begin with it. Many recent studies have revealed that U. The mathematical education they received, both as K students and in teacher preparation, has not provided them with appropriate or sufficient opportunities to learn mathematics. As a result of that education, teachers may know the facts and procedures that they teach but often have a relatively weak understanding of the conceptual basis for that knowledge. Many have difficulty clarifying mathematical ideas or solving problems that involve more than routine calculations. Many have little appreciation of the ways in which mathematical knowledge is generated or justified. Preservice teachers, for example, have repeatedly been shown to be quite willing to accept a series of instances as proving a mathematical generalization. Although teachers may understand the mathematics they teach in only a superficial way, simply taking more of the standard college mathematics courses does not appear to help matters. The evidence on this score has been consistent, although the reasons have not been adequately explored. For example, a study of prospective secondary mathematics teachers at three major institutions showed that, although they had completed the upper-division college mathematics courses required for the mathematics major, they had only a cursory understanding of the concepts underlying elementary mathematics. For the most part, the results have been disappointing: Most studies have failed to find a strong relationship between the two. Many studies, however, have relied on crude measures of these variables. The measure of teacher knowledge, for example, has often been the number of mathematics courses taken or other easily documented data from college Page Share Cite Suggested Citation: Such measures do not provide an accurate index of the specific mathematics that teachers know or of how they hold that knowledge. Teachers may have completed their courses successfully without achieving mathematical proficiency. Or they may have learned the mathematics but not know how to use it in their teaching to help students learn. They may have learned mathematics that is not well connected to what they teach or may not know how to connect it. The empirical literature suggests that this belief needs drastic modification and in fact suggests that once a teacher reaches a certain level of understanding of the subject matter, then further understanding contributes nothing to student achievement. Fourth graders taught by teachers who majored in mathematics education or in education tended to outperform those whose teachers majored in a field other than education. That crude measures of teacher knowledge, such as the number of mathematics courses taken, do not correlate positively with student performance data, supports the need to study more closely the nature of the mathematical knowledge needed to teach and to measure it more sensitively. The research, however, does suggest that proposals to improve mathematics instruction by simply increasing the number of mathematics courses required of teachers are not likely to be successful. As we discuss in the sections that follow, courses that reflect a serious examination of the nature of the mathematics that teachers use in the practice of teaching do have some promise of improving student performance. Teachers need to know mathematics in ways that enable them to help students learn. The specialized knowledge of mathematics that they need is different from the mathematical content contained in most college mathematics courses, which are principally designed for those whose professional uses of mathematics will be in mathematics, science, and other technical fields. Why does this difference matter in considering the mathematical education of teachers? First, the topics taught in upper-level mathematics courses are often remote from the core content of the K curriculum. Although the abstract mathematical ideas are connected, of course, basic algebraic concepts or elementary geometry are not what prospective teachers study in a course in advanced calculus or linear algebra. Second, college mathematics courses do not provide students with opportunities to learn either multiple representations of mathematical ideas or the ways in which different representations relate to one another. Advanced courses do not emphasize the conceptual underpinnings of ideas needed by teachers whose uses of mathematics are to help others learn mathematics. While this approach is important for the education of mathematicians and scientists, it is at odds with the kind of mathematical study needed by teachers. Consider the proficiency teachers need with algorithms. The power of computational algorithms is that they allow learners to calculate without having to think deeply about the steps in the calculation or why the calculations work. Over time, people tend to forget the reasons a procedure works or what is entailed in understanding or justifying a particular algorithm. Because the algorithm has become so automatic, it is difficult to step back and consider what is needed to explain it to someone who does not understand. Most advanced mathematics classes engage students in taking ideas they have already learned and using them to construct increasingly powerful and abstract concepts and methods. Once theorems have been proved, they can be used to prove other theorems. It is not necessary to go back to foundational concepts to learn more advanced ideas. Teaching, however, entails reversing the direction followed in learning advanced mathematics. In helping students learn, teachers must take abstract ideas and unpack them in ways that make the basic underlying concepts visible. For adults, division is an operation on numbers. She wants to put 6 cookies on each plate. How many plates will she need? He wants to put all the cookies on 6 plates. If he puts the same number of cookies on each plate, how many cookies will he put on each plate? These two problems correspond to the measurement and sharing models of division, respectively, that were discussed in chapter 3. Young children using counters solve the first problem by putting 24 counters in piles of 6 counters each. They solve the second by partitioning the 24 counters into 6 groups. In the first case the answer is the number of groups; in the second, it is the number in each group. Until the children are much older, they are not aware that, abstractly, the two solutions are equivalent. Teachers need to see that equivalence so that they can understand and anticipate the difficulties children may have with division. To understand the sense that children are making of arithmetic problems, teachers must understand the distinctions children are making among those problems and how the distinctions might be reflected in how the children think about the problems. The different semantic contexts for each of the operations of arithmetic is not a common topic in college mathematics courses, yet it is essential for teachers to know those contexts and be able to use their knowledge in instruction. The division example illustrates a different way of thinking about the content of courses for teachersâ€”a way that can make those courses more relevant to the teaching of school mathematics. Teachers are unlikely to be able to provide an adequate explanation of concepts they do not understand, and they can hardly engage their students in productive conversations about multiple ways to solve a problem if they themselves can only solve it in a single way. Most of the investigations have been case studies, almost all involving fewer than 10 teachers, and most only one to three teachers. Not surprisingly, these teachers gave the students little assistance in developing an understanding of what they were doing. The teacher also needs to be sensitive to the unique ways of learning, thinking about, and doing mathematics that the student has developed. Each student can be seen as located on a path through school mathematics, equipped with strengths and weaknesses, having developed his or her own approaches to mathematical tasks, and capable of contributing to and profiting from each lesson in a distinctive way. Teachers also need a general knowledge of how students thinkâ€”the approaches that are typical for students of a given age and background, their common conceptions and misconceptions, and the likely sources of those ideas. We have described some of those progressions in chapters 6 through 8. From the many examples of misconceptions to which teachers need to be sensitive, we have chosen one: Children can develop this impression because that is how the notation is often described in the elementary school curriculum and most of their practice exercises fit that pattern. Knowledge of Classroom Practice Knowing classroom practice means knowing what is to be taught and how to plan, conduct, and assess effective lessons on that mathematical content. We have discussed these matters in chapter 9. In the sections that follow, we consider how to develop an integrated corpus of knowledge of the types discussed in this section. First, however, we need to clarify our stance on the relation between knowledge and practice.

Chapter 5 : Elementary and Middle School Mathematics: Teaching Developmentally by John A. Van de Wa

Our mission is to promote student achievement and preparation for global competitiveness by fostering educational excellence and ensuring equal access.

This message has been conveyed in a number of ways: Researchers in mathematics education are in the process of accumulating a persuasive body of evidence that supports the use of manipulative materials in the mathematics classroom. In view of this, it is perplexing that relatively few programs incorporate a substantive experimental component while so many others concentrate merely on completing the pages in the ubiquitous commercially produced textbooks and workbooks. This chapter will discuss the theoretical rationale for using manipulative materials in the classroom, provide a summary of what is already known about the impact of manipulative materials on mathematics learning, discuss barriers to their use, and suggest directions for their use in future research. Whether it was discovered by or has been created by mankind is perhaps a philosophical point and need not concern us here; but the fact is that it exists, and it is extremely useful in describing and predicting events in the world around us. How then is it so useful if it exists "independent of us personally and of the world outside? It does this by creating abstract structures that have properties or attributes similar to its real-world counterpart. We can do this because we know the two systems "behave" in the same manner and because we know that an operation in one system will have its counterpart in the other. This can be depicted as shown in Figure Lesh has suggested that manipulative materials can be effectively used as an intermediary between the real world and the mathematical world. He contends that such use would tend to promote problem-solving ability by providing a vehicle through which children can model real-world situations. The use of manipulative materials concrete models in this manner is thought to be more abstract than the actual situation yet less abstract than the formal symbols. Figure illustrates the revised model. It should be noted that this expanded use departs from the more traditional classroom technique wherein manipulatives have been used to teach children how to calculate using the four arithmetic operations. Relying on the model depicted in Figure , we find it possible to determine the total number of milk containers needed by the three first-grade classes in Main Elementary School by adding the numbers twenty-four, twenty-seven, and twenty-five. It is not necessary to pair each child with a milk container in order to find the correct total because the abstract system of addition is structurally similar to the problem in question and therefore can be used as a model for it. More important, this abstract system is structurally similar to all physical situations where a sum corresponding to the union of a number of disjoint sets is required. It now becomes possible to utilize this more abstract and admittedly less cumbersome system to make conclusions about the more concrete and awkward system. Recall that the problem originated in the real world and was concerned with identifying a one-to-one correspondence between the number of individuals and a like number of milk containers. The problem was then changed into a more suitable, though admittedly more abstract, format; and from a practical standpoint, a considerably more manageable format. It is important to note that this transformation of the problem situation preserved all the important structural aspects of that situation. To complete the problem, the numbers are added, a sum is generated, and conclusions are related to the real-world situation at Main Elementary School. We have come full cycle see Figure The structural similarity between these two systems is known as an isomorphism. It is an extremely important concept in mathematics, for if any two systems can be shown to be isomorphic to one another, it becomes possible to work in the simpler and more available system and transfer all conclusions to the less accessible one. In reality, complete isomorphisms are never really established between an abstract concept and a set of physical materials or a real-world situation. The extent to which the partial isomorphism approximates the concept is the extent to which the more accessible structure is useful in teaching the concept. The fact that some sets of manipulative materials are better than others for teaching a particular concept attests to this. Number is an abstraction. No one has ever seen a number and no one ever will. We see illustrations of this idea everywhere, but we do not see the idea itself. In a similar way the symbol "2" is used to elicit a whole series of recollections and experiences that we have had entailing the concept of two, but the squiggly line 2 in and of itself is not the concept. How then do we teach children about the concept of number if as indicated it is a total abstraction? The answer is very much related to the concept of an isomorphism. For if a parallel structure that was more accessible and perhaps manipulable could be identified having the same properties as the set of whole numbers, then it would be possible to operate within this more accessible and isomorphic structure and subsequently to make conclusions about the more abstract system of number. This is precisely what happens. Sometimes these artificially constructed systems are called interpretations or embodiments of a concept. Some examples of these partial isomorphisms are using counters or sets of objects to represent the counting numbers a discrete model , using lengths such as number lines or Cuisenaire rods to represent the set of real numbers a continuous model , and using the area of a rectangle to represent the multiplication of two whole numbers or fractions. Manipulative materials may now be viewed simply as isomorphic structures that represent the more abstract mathematical notions we wish to have children learn. Each represents the cognitive viewpoint of learning, a position that differs substantially from the connectionist theories that were predominant in educational psychology during the first part of the twentieth century. Modern cognitive psychology places great emphasis on the process dimension of the learning process and is at least as concerned with "how" children learn as with "what", it is they learn. Emphasis is placed, therefore, on the interrelationships between parts as well as the relationship between parts and whole. Each of these men subscribes to a basic tenet of Gestalt psychology, namely that the whole is greater than the sum of its parts. Each suggests that the learning of large conceptual structures is more important than the mastery of large collections of isolated bits of information. Learning is thought to be intrinsic and, therefore, intensely personal in nature. It is the meaning that each individual attaches to an experience which is important. It is generally felt that the degree of meaning is maximized when individuals are allowed and encouraged to interact personally with various aspects of their environment. This, of course, includes other people. I t is the physical action on the part of the child that contributes to her or his understanding of the ideas encountered. Proper use of manipulative materials could be used to promote the broad goals alluded to above. I will discuss each of these men more fully since each has made distinct contributions to a coherent rationale for the use of manipulative materials in the learning of mathematical concepts. Piaget has provided numerous insights into the development of human intelligence, ranging from the random responses of the young infant to the highly complex mental operations inherent in adult abstract reasoning. He has established the framework within which a vast amount of research has been conducted, particularly within the past two decades. In his book The Psychology of Intelligence , Piaget formally develops the stages of intellectual development and the way they are related to the development of cognitive structures. His theory of intellectual development views intelligence as an evolving phenomenon occurring in identifiable stages having a constant order. The age at which children attain and progress through these stages is variable and depends on factors such as physiological maturation, the degree of meaningful social or educational transmission, and the nature and degree of relevant intellectual and psychological experiences. This involves the complementary processes of assimilation fitting new situations into existing psychological frameworks and accommodation modification of behavior by developing or evolving new cognitive structures. Thus the development of intelligence is viewed by Piaget as a dynamic, nonstatic evolution of newer and more complex mental structures. Piaget has argued persuasively that children are not little adults and therefore cannot be treated as such. While it is true that when children reach adolescence their need for concrete experiences is somewhat reduced because of the evolution of new and more sophisticated intellectual schemas, it is not true that this dependence is eliminated. The kinds of thought processes so characteristic of the stage of concrete operations are in fact utilized at all developmental levels beyond the ages of seven or eight. Piaget has emphasized the important role that social interaction plays in both the rate and quality with which intelligence develops. I t would be impossible to incorporate the essence of these ideas into a mathematics program that relies primarily or exclusively on the printed page for its direction and "activities. Dienes and Bruner, while generally espousing the views of Piaget, have made contributions to the cognitive view of mathematics learning that are distinctly their own. The work of these two men lends additional support to this point of view. Dienes Unlike Piaget, Dienes has concerned himself exclusively with mathematics learning; yet like Piaget, his major message is concerned with providing a justification for active student involvement in the learning process. Such involvement routinely involves the use of a vast amount of concrete material. Rejecting the position that mathematics is to be learned primarily for utilitarian or materialistic reasons, Dienes sees mathematics as an art form to be studied for the intrinsic value of the subject itself. Dienes has expressed concern with many aspects of the status quo, including the restricted nature of mathematical content considered, the narrow focus of program objectives, the overuse of large-group instruction, the debilitating nature of the punishment- reward system grading , and the limited dimension of tile instructional methodology used in most classrooms. Each will be discussed briefly and its implications noted. The reader will notice large-scale similarities to the work of Piaget. This principle suggests that true understanding of a new concept is an evolutionary process involving the learner in three temporally ordered stages. The first stage is the preliminary or play stage, and it involves the learner with the concept in a relatively unstructured but not random manner. Following the informal exposure afforded by the play stage, more structured activities are appropriate, and this is the second stage. It is here that the child is given experiences that are structurally similar isomorphic to the concepts to be learned. The third stage is characterized by the emergence of the mathematical concept with ample provision for reapplication to the real world. This cyclical pattern can be depicted as shown in Figure The completion of this cycle is necessary before any mathematical concept becomes operational for the learner. In subsequent work Dienes elaborated upon this process and referred to it as a learning cycle Dienes , Dienes and Golding The dynamic principle establishes a general framework within which learning of mathematics can occur. The remaining components should be considered as existing within this framework. The Perceptual Variability Principle. This principle suggests that conceptual learning is maximized when children are exposed to a concept through a variety of physical contexts or embodiments. The experiences provided should differ in outward appearance while retaining the same basic conceptual structure. The provision of multiple experiences not the same experience many times , using a variety of materials, is designed to promote abstraction of the mathematical concept. When a child is given opportunities to see a concept in different ways and under different conditions, he or she is more likely to perceive that concept irrespective of its concrete embodiment. For example, the regrouping procedures used in the process of adding two numbers is independent of the type of materials used. We could therefore use tongue depressors, chips, and abacus or multibase arithmetic blocks to illustrate this process. When exposed to a number of seemingly different tasks that are identical in structure, children will tend to abstract the similar elements from their experiences. It is not the performance of anyone of the individual tasks that is the mathematical abstraction but the ultimate realization of their similarity. The Mathematical Variability Principle. This principle suggests that the generalization of a mathematical concept is enhanced when the concept is perceived under conditions wherein variables irrelevant to that concept are systematically varied while keeping the relevant variables constant. For example, if one is interested in promoting an understanding of the parallelogram, this principle suggests that it is desirable to vary as many of the irrelevant attributes as possible. In this example, the size of angles, the length of sides, the position on the paper should be varied while keeping the relevant attribute-opposite sides parallel-intact. Dienes suggests that the two variability principles be used in concert with one another since they are designed to promote the complementary processes of abstraction and generalization, both of which are crucial aspects of conceptual development. Dienes identifies two kinds of thinkers: This principle states simply that "construction should always precede analysis. According to Dienes, these experiences carefully selected by the teacher form the cornerstone upon which all mathematics learning is based. At some future time, attention will be directed toward the analysis of what has been constructed; however, Dienes points out that it is not possible to analyze what is not yet there in some concrete form.

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