

Chapter 1 : What Floats Your Boat? - Lesson - TeachEngineering

Archimedes' principle is named after Archimedes of Syracuse, who first discovered this law in B.C. For objects, floating and sunken, and in gases as well as liquids (i.e. a fluid), Archimedes' principle may be stated thus in terms of forces.

Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object. In simple terms, the principle states that the buoyant force on an object is going to be equal to the weight of the fluid displaced by the object, or the density of the fluid multiplied by the submerged volume. Thus, among completely submerged objects with equal masses, objects with greater volume have greater buoyancy. Suppose that when the rock is lowered into water, it displaces water of weight 3 newtons. The force it then exerts on the string from which it hangs would be 10 newtons minus the 3 newtons of buoyant force: Buoyancy reduces the apparent weight of objects that have sunk completely to the sea floor. It is generally easier to lift an object up through the water than it is to pull it out of the water. The density of the immersed object relative to the density of the fluid can easily be calculated without measuring any volumes: If you drop wood into water buoyancy will keep it afloat. A helium balloon in a moving car. The corresponding equilibrium equation is: In this case the stress tensor is proportional to the identity tensor: Using this the above equation becomes: Let the z-axis point downward. Any object with a non-zero vertical depth will have different pressures on its top and bottom, with the pressure on the bottom being greater. This difference in pressure causes the upward buoyancy forces. The buoyant force exerted on a body can now be calculated easily, since the internal pressure of the fluid is known. The force exerted on the body can be calculated by integrating the stress tensor over the surface of the body which is in contact with the fluid: The magnitude of buoyant force may be appreciated a bit more from the following argument. Consider any object of arbitrary shape and volume V surrounded by a liquid. The force the liquid exerts on an object within the liquid is equal to the weight of the liquid with a volume equal to that of the object. This force is applied in a direction opposite to gravitational force, that is of magnitude: If this volume of liquid is replaced by a solid body of the exact same shape, the force the liquid exerts on it must be exactly the same as above. An object whose weight exceeds its buoyancy tends to sink. Calculation of the upwards force on a submerged object during its accelerating period cannot be done by the Archimedes principle alone; it is necessary to consider dynamics of an object involving buoyancy. Once it fully sinks to the floor of the fluid or rises to the surface and settles, Archimedes principle can be applied alone. For a floating object, only the submerged volume displaces water. For a sunken object, the entire volume displaces water, and there will be an additional force of reaction from the solid floor. For this reason, a ship may display a Plimsoll line. It can be the case that forces other than just buoyancy and gravity come into play. This is the case if the object is restrained or if the object sinks to the solid floor. An object which tends to float requires a tension restraint force T in order to remain fully submerged. An object which tends to sink will eventually have a normal force of constraint N exerted upon it by the solid floor. The constraint force can be tension in a spring scale measuring its weight in the fluid, and is how apparent weight is defined. If the object would otherwise float, the tension to restrain it fully submerged is: If an object which usually sinks is submerged suspended via a cord from a balance pan, the reference object on the other dry-land pan of the balance will have mass: If the fluid density is greater than the average density of the object, the object floats; if less, the object sinks. Another possible formula for calculating buoyancy of an object is by finding the apparent weight of that particular object in the air calculated in Newtons , and apparent weight of that object in the water in Newtons. To find the force of buoyancy acting on the object when in air, using this particular information, this formula applies: For this reason, the weight of an object in air is approximately the same as its true weight in a vacuum. The buoyancy of air is neglected for most objects during a measurement in air because the error is usually insignificant typically less than 0. For example, floating objects will generally have vertical stability, as if the object is pushed down slightly, this will create a greater buoyant force, which, unbalanced by the weight force, will push the object back up. Rotational stability is of great importance to floating vessels. Given a small angular displacement, the vessel may return to its original position stable , move away from its original position unstable , or remain where it is neutral.

Rotational stability depends on the relative lines of action of forces on an object. The upward buoyant force on an object acts through the center of buoyancy, being the centroid of the displaced volume of fluid. The weight force on the object acts through its center of gravity. As an airship rises in the atmosphere, its buoyancy decreases as the density of the surrounding air decreases. As a submarine expels water from its buoyancy tanks by pumping them full of air it rises because its volume is constant the volume of water it displaces if it is fully submerged as its weight is decreased. If, however, its compressibility is greater, its equilibrium is then unstable, and it rises and expands on the slightest upward perturbation, or falls and compresses on the slightest downward perturbation. Submarines rise and dive by filling large tanks with seawater. To dive, the tanks are opened to allow air to exhaust out the top of the tanks, while the water flows in from the bottom. Once the weight has been balanced so the overall density of the submarine is equal to the water around it, it has neutral buoyancy and will remain at that depth. Normally, precautions are taken to ensure that no air has been left in the tanks. If air were left in the tanks and the submarine were to descend even slightly, the increased pressure of the water would compress the remaining air in the tanks, reducing its volume. Since buoyancy is a function of volume, this would cause a decrease in buoyancy, and the submarine would continue to descend. The height of a balloon tends to be stable. The average density of the balloon decreases less, therefore, than that of the surrounding air. A rising balloon tends to stop rising. Similarly, a sinking balloon tends to stop sinking. A density column containing some common liquids and solids. Food coloring was added to rubbing alcohol and water for visibility. If the weight of an object is less than the weight of the displaced fluid when fully submerged, then the object has an average density that is less than the fluid and when fully submerged will experience a force buoyancy greater than its own weight. If the fluid has a surface, such as water in a lake or the sea, the object will float and settle at a level where it displaces the same weight of fluid as the weight of the object. If the object is immersed in the fluid, such as a submerged submarine or air in a balloon, it will tend to rise. If the object has exactly the same density as the fluid, then its buoyancy equals its weight. It will remain submerged in the fluid, but it will neither sink nor float, although a disturbance in either direction will cause it to drift away from its position. An object with a higher average density than the fluid will never experience more buoyancy than weight and it will sink. A ship will float even though it may be made of steel which is much denser than water, because it encloses a volume of air which is much less dense than water, and the resulting shape has an average density less than that of the water. In its simple form, it applies when the object is not accelerating relative to the fluid. To examine the case when the object is accelerated by buoyancy and gravity, the fact that the displaced fluid itself has inertia as well must be considered. All acceleration measures are relative to the reference frame of the undisturbed background fluid. The displaced parcel of fluid is indicated as the dark blue rectangle, and the buoyant solid object is indicated as the gray object. The acceleration vectors a in this visual depict a positively buoyant object which naturally accelerates upward, and upward acceleration of the object is our sign convention. The following simple formulation makes the assumption of slow speeds such that drag and viscosity are not significant. It is difficult to carry out such an experiment in practice with speeds close to zero, but if measurements of acceleration are made as quickly as possible after release from rest, the equations below give a good approximation to the acceleration and the buoyancy force. In many situations turbulence will introduce other forces that are much more complex to calculate. Thus B reduces to mg and the acceleration is zero. Similarly, if the fluid is much denser than the object, then B approaches $2mg$ and the upward acceleration is approximately g .

righting moment, and the object becomes unstable. It is possible to shift from positive to negative or vice versa more than once during a heeling disturbance, and many shapes are stable in more than one position. Compressible fluids and objects[edit] This section does not cite any sources. Please help improve this section by adding citations to reliable sources. Unsourced material may be challenged and removed. As an airship rises in the atmosphere, its buoyancy decreases as the density of the surrounding air decreases. In contrast, as a submarine expels water from its buoyancy tanks, it rises because its volume is constant the volume of water it displaces if it is fully submerged while its mass is decreased. If, however, its compressibility is greater, its equilibrium is then unstable , and it rises and expands on the slightest upward perturbation, or falls and compresses on the slightest downward perturbation. To dive, the tanks are opened to allow air to exhaust out the top of the tanks, while the water flows in from the bottom. Once the weight has been balanced so the overall density of the submarine is equal to the water around it, it has neutral buoyancy and will remain at that depth. Most military submarines operate with a slightly negative buoyancy and maintain depth by using the "lift" of the stabilizers with forward motion. As a balloon rises it tends to increase in volume with reducing atmospheric pressure, but the balloon itself does not expand as much as the air on which it rides. The average density of the balloon decreases less than that of the surrounding air. The weight of the displaced air is reduced. A rising balloon stops rising when it and the displaced air are equal in weight. Similarly, a sinking balloon tends to stop sinking. Divers[edit] Underwater divers are a common example of the problem of unstable buoyancy due to compressibility. The diver typically wears an exposure suit which relies on gas-filled spaces for insulation, and may also wear a buoyancy compensator , which is a variable volume buoyancy bag which is inflated to increase buoyancy and deflated to decrease buoyancy. The desired condition is usually neutral buoyancy when the diver is swimming in mid-water, and this condition is unstable, so the diver is constantly making fine adjustments by control of lung volume, and has to adjust the contents of the buoyancy compensator if the depth varies. This section does not cite any sources. If the weight of an object is less than the weight of the displaced fluid when fully submerged, then the object has an average density that is less than the fluid and when fully submerged will experience a buoyancy force greater than its own weight. If the fluid has a surface, such as water in a lake or the sea, the object will float and settle at a level where it displaces the same weight of fluid as the weight of the object. If the object is immersed in the fluid, such as a submerged submarine or air in a balloon, it will tend to rise. If the object has exactly the same density as the fluid, then its buoyancy equals its weight. It will remain submerged in the fluid, but it will neither sink nor float, although a disturbance in either direction will cause it to drift away from its position. An object with a higher average density than the fluid will never experience more buoyancy than weight and it will sink. A ship will float even though it may be made of steel which is much denser than water , because it encloses a volume of air which is much less dense than water , and the resulting shape has an average density less than that of the water.

Chapter 3 : Archimedes, The Sand-Reckoner | Quintus Curtius

Archimedes principle: So, for a floating object on a liquid, the weight of the displaced liquid is the weight of the object. Thus, only in the special case of.

Path to this page: In many ways, this is what mathematics is about. Take the most ordinary objects, look at them through the infinitely powered magnifying glass that mathematics is "and you come to see infinity in one of its many indeed, infinite forms. Draw a circle " what is the ratio of its circumference to its diameter? To simplify things a bit, this is because the number Pi has infinitely many, endlessly varying digits. Draw now a square, presumably a simpler object. What is the ratio of the side to the diagonal? Once again, one needs infinity, of a somewhat different kind, in order to express this. And now draw just a line, any line. How many points does it contain? Right again " infinitely many. Which kind of infinity? Curiously enough, mathematicians do not know the answer to this. The simplest line, with the simplest question, already brings us to an enigma of infinity that mathematics still cannot solve. The enigma of infinity has fed into all mathematical progress. Given that there are infinitely many prime numbers, are there ways in which we can characterize how many precisely they are? Much of number theory emerges out of this question. And even more fundamentally: He measures spiral lines, spheroids, conoids, spheres and parabolas, again and again extending the ways by which measurement may be affected on curves. By potential infinity something such as the following is meant. Suppose you go into an auction, representing a buyer who mysteriously guarantees you that he has limitless funds, that is, he guarantees you that, however much the opponents are offering, you are allowed to beat their offers. They offer one million " and you offer two. They offer a billion " and you a trillion. However much they put up, you may put up more. You are the representative, then, of potential infinity. Why is this infinity merely potential? Because you are always making a finite commitment. In other words, you know that, if the auction is ever to end, then it will end with a finite commitment " however large that finite commitment may be. It may be a million, a trillion, or it may be the number 1 followed by a trillion zeroes, but it will always be some finite number. Potential infinity, then, may be viewed as an endlessly extendible, and yet forever finite, magnitude. Not so actual infinity. By actual infinity we mean something such as, say, the number of points on a line. If your buyer would have authorized you to offer as your price, in that auction, a dollar for each point there is on a line, then he would have authorized you to make an actually infinite bid. This is a very high bid indeed. After all, even the number 1, followed by a trillion zeroes, is still dwarfed by actual infinity. It is a curious bid, too. If it takes you, say, a minute to pass a suitcase with a million dollars, than it will take you all of eternity to pay your bid actually infinite bid. Actual infinity actually never ends. And for this reason many philosophers, throughout the ages, have doubted its very existence. The established historical wisdom on infinity and the history of mathematics, then, used to go something like this. At first came the Greeks, who preferred above all to have precise, rigorous proofs. For this reason they have completely avoided the concept of actual infinity, concentrating instead on potential infinity whose rigorous treatment was perfected by Archimedes in his measurement of curves. Next came the early modern mathematicians, who wanted to get results come what may. To obtain general result concerning curves, they have brought in the notion of actual infinity " paying the price that their mathematics was less rigorous and precise compared to that of the Greeks. Along came the mathematicians of the 19th and 20th centuries, slowly and laboriously building up a new kind of mathematics, where the concept of actual infinity is meticulously built up so as to have the same kind of rigorous foundations that the Greeks have provided for potential infinity. They have changed no less than our understanding of how western mathematicians came to handle infinity. To be sure, the Method was always considered, ever since the discovery of the Palimpsest in , as the most significant contribution made by the Palimpsest to our knowledge of the history of mathematics. And it survives on the Palimpsest alone. In it, Archimedes sets out to solve many separate problems using a variety of techniques, most often one involving a striking combination of physics and of mathematics. For instance, two geometrical objects are considered simultaneously, say a triangle and a parabolic segment. Then Archimedes shows how each line in the triangle balances a line in the parabolic segment around a given

fulcrum, so that it follows that the triangle as a whole, and the parabolic segment as a whole, balance as well around that fulcrum. So much was known since This was striking indeed but even more so was the discovery made in In proposition 14 of the Method, it turns out, as Archimedes is measuring the volume of a cylindrical segment, he makes systematic reference which previous readers could not decipher, in part because this was so unexpected to actual infinity itself. This pushes the mathematical use of actual infinity nearly some 2, years back in time. The curious twist is this: This the number of points on a line is still the big unsolved riddle of mathematics. The enigma of infinity still haunts us, staring us, as it were, at the face even from the pages of the Palimpsest.

Chapter 4 : Archimedes Facts

Archimedes' principle states that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces and acts in the upward direction at the center of mass of the displaced fluid.

He was the son of an astronomer and mathematician named Phidias. Aside from that, very little is known about the early life of Archimedes or his family. Some maintain that he belonged to the nobility of Syracuse, and that his family was in some way related to that of Hiero II, King of Syracuse. In the third century BC, Syracuse was a hub of commerce, art and science. As a youth in Syracuse Archimedes developed his natural curiosity and penchant for problem solving. When he had learned as much as he could from his teachers, Archimedes traveled to Egypt in order to study in Alexandria. After his studies in Alexandria, Archimedes returned to Syracuse and pursued a life of thought and invention. Many apocryphal legends record how Archimedes endeared himself to King Hiero II, discovering solutions to problems that vexed the king. The King called upon Archimedes for assistance. When the lower end of the tube was placed into the hull and the handle turned, water was carried up the tube and out of the boat. The Archimedes Screw is still used as a method of irrigation in developing countries. When the crown arrived, King Hiero was suspicious that the goldsmith only used some of the gold, kept the rest for himself and added silver to make the crown the correct weight. Archimedes was asked to determine whether or not the crown was pure gold without harming it in the process. Archimedes was perplexed but found inspiration while taking a bath. The volcanic Mount Etna loomed threateningly over the island, while on all sides the titanic Punic Wars raged between Rome and Carthage. Situated strategically between the two great powers, Sicily naturally became an object of contention. Self preservation demanded that the kings of Syracuse negotiate with the great powers, and as a result the small city-state often found itself allied with one against the other. Such was the case in BC, when pro-Carthaginian factions within the city chose to side with Carthage against Rome. Shortly thereafter, legions of the Roman army sailed to Syracuse and laid siege to the city walls. King Hiero II had anticipated such an eventuality. Before his death in BC, Hiero set Archimedes to work, strengthening the walls of Syracuse and modifying its great stronghold, the Euryelos fortress. Archimedes also constructed war machines to defend Syracuse. When the Romans arrived under the command of the famed general Marcellus, Archimedes was prepared. The Roman historian Polybius relates that Archimedes now made such extensive preparations, both within the city and also to guard against an attack from the sea, that there would be no chance of the defenders being employed in meeting emergencies but that every move of the enemy could be replied to instantly by a counter move. The Death of Archimedes For two years the genius of Archimedes repelled the Romans, enabling the city to survive the lengthy siege. Nevertheless, in BC the forces of Marcellus prevailed and took the city. Marcellus had great respect for Archimedes, and immediately dispatched soldiers to retrieve his foe. Apparently, the great mathematician was unaware that his enemy had stormed the city, so deeply were his attentions focused on a mathematical problem. When a soldier demanded Archimedes accompany him to the quarters of Marcellus he simply refused, and continued his ruminations. The enraged soldier flew upon Archimedes, striking the 75 year-old eccentric dead. Through the medium of geometry, he was able to elucidate the principles for such basic devices as the pulley, the fulcrum and the lever – devices still utilized today. Archimedes is also credited with the discovery of the principle of buoyancy, or the power of a fluid to exert an upward force on a body placed in it. His further research into volume and density was fundamental to the development of theories of hydrostatics-the branch of physics dealing with liquids at rest. But an essential point is this: One was last heard of in 212 BC, a second was last heard of in the 15th century, and the third is The Archimedes Palimpsest, now at The Walters Art Museum in Baltimore, and the subject of this website. Because this is just the start of a fascinating story.

Chapter 5 : On Floating Bodies - Wikipedia

In order for Archimedes' principle to be used alone, the object in question must be in equilibrium (the sum of the forces on the object must be zero), therefore; and therefore showing that the depth to which a floating object will sink, and the volume of fluid it will displace, is independent of the gravitational field regardless of geographic.

See Article History Fluid mechanics, science concerned with the response of fluids to forces exerted upon them. It is a branch of classical physics with applications of great importance in hydraulic and aeronautical engineering , chemical engineering , meteorology , and zoology. The most familiar fluid is of course water , and an encyclopaedia of the 19th century probably would have dealt with the subject under the separate headings of hydrostatics, the science of water at rest, and hydrodynamics, the science of water in motion. The foundations of hydrodynamics, on the other hand, were not laid until the 18th century when mathematicians such as Leonhard Euler and Daniel Bernoulli began to explore the consequences, for a virtually continuous medium like water, of the dynamic principles that Newton had enunciated for systems composed of discrete particles. Their work was continued in the 19th century by several mathematicians and physicists of the first rank, notably G. Stokes and William Thomson. By the end of the century explanations had been found for a host of intriguing phenomena having to do with the flow of water through tubes and orifices, the waves that ships moving through water leave behind them, raindrops on windowpanes, and the like. There was still no proper understanding, however, of problems as fundamental as that of water flowing past a fixed obstacle and exerting a drag force upon it; the theory of potential flow, which worked so well in other contexts , yielded results that at relatively high flow rates were grossly at variance with experiment. This problem was not properly understood until , when the German physicist Ludwig Prandtl introduced the concept of the boundary layer see below Hydrodynamics: Boundary layers and separation. Since that time, the flow of air has been of as much interest to physicists and engineers as the flow of water, and hydrodynamics has, as a consequence, become fluid dynamics. The term fluid mechanics , as used here, embraces both fluid dynamics and the subject still generally referred to as hydrostatics. One other representative of the 20th century who deserves mention here besides Prandtl is Geoffrey Taylor of England. Taylor remained a classical physicist while most of his contemporaries were turning their attention to the problems of atomic structure and quantum mechanics , and he made several unexpected and important discoveries in the field of fluid mechanics. The richness of fluid mechanics is due in large part to a term in the basic equation of the motion of fluids which is nonlinearâ€™i. It is characteristic of systems described by nonlinear equations that under certain conditions they become unstable and begin behaving in ways that seem at first sight to be totally chaotic. In the case of fluids, chaotic behaviour is very common and is called turbulence. Mathematicians have now begun to recognize patterns in chaos that can be analyzed fruitfully, and this development suggests that fluid mechanics will remain a field of active research well into the 21st century. For a discussion of the concept of chaos , see physical science, principles of. Fluid mechanics is a subject with almost endless ramifications, and the account that follows is necessarily incomplete. Some knowledge of the basic properties of fluids will be needed; a survey of the most relevant properties is given in the next section. For further details, see thermodynamics and liquid. Basic properties of fluids Fluids are not strictly continuous media in the way that all the successors of Euler and Bernoulli have assumed, for they are composed of discrete molecules. The molecules, however, are so small and, except in gases at very low pressures, the number of molecules per millilitre is so enormous that they need not be viewed as individual entities. There are a few liquids, known as liquid crystals, in which the molecules are packed together in such a way as to make the properties of the medium locally anisotropic , but the vast majority of fluids including air and water are isotropic. A word perhaps is needed about the difference between gases and liquids , though the difference is easier to perceive than to describe. In gases the molecules are sufficiently far apart to move almost independently of one another, and gases tend to expand to fill any volume available to them. In liquids the molecules are more or less in contact, and the short-range attractive forces between them make them cohere; the molecules are moving too fast to settle down into the ordered arrays that are characteristic of solids, but not so fast that they can fly apart. Thus, samples of liquid can exist

as drops or as jets with free surfaces, or they can sit in beakers constrained only by gravity, in a way that samples of gas cannot. Such samples may evaporate in time, as molecules one by one pick up enough speed to escape across the free surface and are not replaced. The lifetime of liquid drops and jets, however, is normally long enough for evaporation to be ignored. There are two sorts of stress that may exist in any solid or fluid medium, and the difference between them may be illustrated by reference to a brick held between two hands. If the holder moves his hands toward each other, he exerts pressure on the brick; if he moves one hand toward his body and the other away from it, then he exerts what is called a shear stress. A solid substance such as a brick can withstand stresses of both types, but fluids, by definition, yield to shear stresses no matter how small these stresses may be. This property, about which more will be said later, is a measure of the friction that arises when adjacent layers of fluid slip over one another. These three quantities are linked together by what is called the equation of state for the fluid. For gases at low pressures the equation of state is simple and well known. It is where R is the universal gas constant 8. For other fluids knowledge of the equation of state is often incomplete. When an element of fluid is compressed, the work done on it tends to heat it up. For liquids the ratio between the isothermal and adiabatic compressibilities is much closer to unity. The molar specific heat is the amount of heat required to raise the temperature of one mole through one degree. This is greater if the substance is allowed to expand as it is heated, and therefore to do work, than if its volume is fixed. The principal molar specific heats, C_P and C_V , refer to heating at constant pressure and constant volume, respectively, and for air, C_P is about 3. Solids can be stretched without breaking, and liquids, though not gases, can withstand stretching, too. Water owes its high ideal strength to the fact that rupture involves breaking links of attraction between molecules on either side of the plane on which rupture occurs; work must be done to break these links. However, its strength is drastically reduced by anything that provides a nucleus at which the process known as cavitation formation of vapour- or gas-filled cavities can begin, and a liquid containing suspended dust particles or dissolved gases is liable to cavitate quite easily. Work also must be done if a free liquid drop of spherical shape is to be drawn out into a long thin cylinder or deformed in any other way that increases its surface area. Here again work is needed to break intermolecular links. The surface of a liquid behaves, in fact, as if it were an elastic membrane under tension, except that the tension exerted by an elastic membrane increases when the membrane is stretched in a way that the tension exerted by a liquid surface does not. Surface tension is what causes liquids to rise up capillary tubes, what supports hanging liquid drops, what limits the formation of ripples on the surface of liquids, and so on. Applied to the atmosphere, equation would imply that the pressure falls to zero at a height of about 10 kilometres. Compressible flow in gases. Differential manometers Instruments for comparing pressures are called differential manometers, and the simplest such instrument is a U-tube containing liquid, as shown in Figure 1A. It is a consequence of that Figure 1: Schematic representations of A a differential manometer, B a Torricellian barometer, and C a siphon. A barometer for measuring the pressure of the atmosphere in absolute terms is simply a manometer in which p_2 is made zero, or as close to zero as is feasible. The barometer invented in the 17th century by the Italian physicist and mathematician Evangelista Torricelli, and still in use today, is a U-tube that is sealed at one end see Figure 1B. It may be filled with liquid, with the sealed end downward, and then inverted. On inversion, a negative pressure may momentarily develop at the top of the liquid column if the column is long enough; however, cavitation normally occurs there and the column falls away from the sealed end of the tube, as shown in the figure. Between the two exists what Torricelli thought of as a vacuum, though it may be very far from that condition if the barometer has been filled without scrupulous precautions to ensure that all dissolved or adsorbed gases, which would otherwise collect in this space, have first been removed. Even if no contaminating gas is present, the Torricellian vacuum always contains the vapour of the liquid, and this exerts a pressure which may be small but is never quite zero. The liquid conventionally used in a Torricelli barometer is of course mercury, which has a low vapour pressure and a high density. The high density means that h is only about millimetres; if water were used, it would have to be about 10 metres instead. Figure 1C illustrates the principle of the siphon. The top container is open to the atmosphere, and the pressure in it, p_2 , is therefore atmospheric. If the bottom container is also open to the atmosphere, then equilibrium is clearly impossible; the weight of the liquid column prevails and causes the liquid to flow downward. The siphon operates only as

long as the column is continuous; it fails if a bubble of gas collects in the tube or if cavitation occurs. Cavitation therefore limits the level differences over which siphons can be used, and it also limits to about 10 metres the depth of wells from which water can be pumped using suction alone. As Archimedes must have realized, there is no need to prove this by detailed examination of the pressure difference between top and bottom. It is obvious because, if the solid body could somehow be removed and if the cavity thereby created could somehow be filled with more fluid instead, the whole system would still be in equilibrium. The extra fluid would, however, then be experiencing the upthrust previously experienced by the solid body, and it would not be in equilibrium unless this were just sufficient to balance its weight. He understood that the pure metal and the alloy would differ in density and that he could determine the density of the crown by weighing it to find its mass and making a separate measurement of its volume. Perhaps the inspiration that struck him in his bath was that one can find the volume of any object by submerging it in liquid in something like a measuring cylinder. If so, he no doubt realized soon afterward that a more elegant and more accurate method for determining density can be based on the principle that bears his name. What that means is that the object does not submerge of its own accord; it has to be pushed downward to make it do so. A hydrometer is an object graduated in such a way that this fraction may be measured. In what orientation an object floats is a matter of grave concern to those who design boats and those who travel in them. A simple example will suffice to illustrate the factors that determine orientation. In each of these diagrams, C is the centre of mass of the prism, and B, a point known as the centre of buoyancy, is the centre of mass of the displaced water. The distributed forces acting on the prism are equivalent to its weight acting downward through C and to the equal weight of the displaced water acting upward through B. In general, therefore, the prism experiences a torque. In Figure 2B the torque is counterclockwise, and so it turns the prism away from 2A and toward 2C. In 2C the torque vanishes because B is now vertically below C, and this is the orientation that corresponds to stable equilibrium. The torque also vanishes in 2A, and the prism can in principle remain indefinitely in that orientation as well; the equilibrium in this case, however, is unstable, and the slightest disturbance will cause the prism to topple one way or the other. In fact, the potential energy of the system, which increases in a linear fashion with the difference in height between C and B, is at its smallest in orientation 2C and at its largest in orientation 2A. To improve the stability of a floating object one should, if possible, lower C relative to B. In the case of a boat, this may be done by redistributing the load inside. Three possible orientations of a uniform square prism floating in liquid of twice its density. The stable orientation is C see text. Surface tension of liquids Of the many hydrostatic phenomena in which the surface tension of liquids plays a role, the most significant is probably capillarity. Consider what happens when a tube of narrow bore, often called a capillary tube, is dipped into a liquid. If the liquid does not wet the tube, the meniscus is convex and depressed through the same distance h see Figure 3. A simple method for determining surface tension involves the measurement of h in one or the other of these situations and the use of equation thereafter. It follows from equations and that the pressure at a point P just below the meniscus differs from the pressure at Q by an amount it is less than the pressure at Q in the case to which Figure 3A refers and greater than the pressure at Q in the other case. Since the pressure at Q is just the atmospheric pressure, it is equal to the pressure at a point immediately above the meniscus. Such a pressure difference is a requirement of equilibrium wherever a liquid surface is curved. If the surface is curved but not spherical, the pressure difference is where r_1 and r_2 are the two principal radii of curvature. If it is cylindrical, one of these radii is infinite, and, if it is curved in opposite directions, then for the purposes of they should be treated as being of opposite sign.

Chapter 6 : The Diagrams as Floating Bodies by Reviel Netz of Stanford University

The work done by Archimedes (ca. B.C.), a Greek mathematician, was wide ranging, some of it leading to what has become integral calculus. He is considered one of the greatest mathematicians of all time. Archimedes probably was born in the seaport city of Syracuse, a Greek colony on the.

Archimedes screw generators are the technology we use. Archimedes Screw History – Traditional use of the screw has been as a pump To reclaim land as with this windmill pump in Holland. To irrigate land – past and present day. To pump fish in the aquaculture industry. Simple, robust, reliable, few parts. There is a coarse intake grate to keep out the very large debris and for safety purposes. Simple weight of the water turns the screw, which turns the gearbox which turns the generator. For example, there is approx. The diameter is increased to accommodate larger flows. The length is increased to accommodate larger heads. The screw needs 1 meter of head as a minimum, and can go up to 10 meters in some cases, but the screw is most likely to be used on heads of less than 5 meters. The screw is classified as a lower head, higher flow technology but the effective higher flow is going to be limited to the diameter of screw that can be transported down the road without incurring special costs for excess height, which is about 4 meters in diameter, thereby limiting the upper flow range of a single screw to around six and a half cubic meters per second approximately. Beyond this a second screw would have to be used to accommodate more flow or extra transportation costs incurred. ASGs have greatest potential at low head sites less than about 5 meters of head , and unlike conventional reaction or impulse turbines, have the potential for maintaining high efficiencies even as the head approaches zero. Each screw is custom made for the watercourse flow and head and this is easily done. Manufacture of the screw takes about a week. We prefer to bypass these extra costs because micro hydro does not have the scale to withstand these types of costs. Usage and refurbishment of existing channels and intakes is possible to reduce site disruption and civil construction as it was at the Fletcher site. We utilized the old existing turbine intake and refurbished it, cleaned out the old channel and simply added an addition on the end. In this case we did not utilize the old mill tail race, but instead turned the screw 90 degrees to have the water exit for the screw enter the watercourse immediately below the weir, in order to eliminate any depleted reach that would have been caused by using the old tail race. All the work on the addition was done on dry ground, and when completed the earth between the screw exit and the watercourse was removed. Cold weather performance of the Archimedes Screw System While the top surface of the water may freeze, the water always flows, and to prevent icing of the screw as it rotates we have a simple air seal arrangement at the intake and at the water exit. At the entrance is a simple concrete slab that extends just below the water and at the water exit, stop logs do the same thing. This prevents cold air from entering the screw chamber and generator room. With the thermal mass of the water and the small amount of heat emitted from the generator, the generator room stays just above freezing without any external heat source. The Fletcher installation worked throughout one of the coldest winters in Ontario history with no icing issues in spite of an uninsulated generator room. Our thermal modelling will tell us when we need to provide additional insulation in the generator room. As installations get larger the generator size relative to the generator room tends to increase therefore freezing is less of a concern, and venting in the summer is a larger issue. Small Watt capacity Archimedes Screw generator of the Open Complete model type sitting on small weir spillway. Technical issues that make the Archimedes Screw generator environmentally attractive. Bottom draw capable – design of the intake can accommodate bottom draw of flow. Lower bearing – is a proprietary water lubricated composite or a proprietary submersible roller bearing assembly that is not grease fed. The lower submersible bearing in the screw at Fletchers is a simple water lubricated composite bearing. Upper bearing – is a simple bearing with an auto lubrication device attached to grease it occasionally, but there is nothing to spill and this bearing is out of the water. Upper gear box – contains food grade oil. The protective coating on the screw is a two part epoxy which after application and hardening the coating is chemically inert. Slow tip speed – The reason that rpm decreases as screw diameter increases is to keep the tip speed the outer edge of the flighting safe for fish. Compressible Rubber bumpers – as additional fish protection, we install rubber bumpers on the leading edge

of the fighting so that it absorbs any impact with a fish entering the screw. We use compressible rubber bumpers as standard practice. Acts like an elevator not a blender” The screw acts like an elevator for the water than enters, not a blender. The water in this bucket remains relatively stationary in the bottom of the screw as the screw turns. Therefore any fish that do enter get lowered from top to bottom with the water. This can be seen in the diagram below. The chamber of water available for fish while being lowered is very large. This can be seen in the examples diagram below Absence of extreme pressures or shear stress associated with more conventional high head turbines” The Archimedes screw does not work based on the same water pressure concept as do traditional turbines, there is no increased water pressure or high rpms associated with water moving through the screw as there is with traditional turbines. In one of the studies the effects of turbulence and shear stress on fish orientation within the screw compartments was assessed using camera footage of fish passing down the screw. No adverse effects were observed and this was deemed to show that the startle response of fish would not be affected by passage down the screw. The result of all of the above is safe downstream fish passage. In another study In order to assess attraction of adult salmonids to the outflow, underwater cameras were positioned to record fish approaching the outflow. Twenty-five fish were seen by the cameras, spending an average of 8 minutes in the outflow area. No fish were seen attempting to jump up the screw. True run of river” the screw only takes water that would have otherwise went over the weir to generate electricity. Flows beyond the screw capacity, continue to flow over the weir. If at any time the screws flow capacity exceeds the watercourse flow, this is when the sluice gate can become active to restrict flow to the screw to maintain the target water level at the weir. Increases dam maintenance by having parties with a vested interest in the Dam structures. Increases flood flow capacity of the site” when installed off to the side of the main weir structure, the screw adds to the flood flow capacity of the site. While the sluice gate on the screw intake can be used to maintain a target water level at the weir to ensure a true run of river system, this could be changed to allow the screw to lower water levels below the weir crest by a certain amount if desired. This would no longer be true run of river, but may be desirable in some situations such as a planned draw down of water levels to allow for flood flows. Hopefully screw installations reduce other forms of generation that are non-renewable. Many low head dams are near or in small communities due to historical origins allowing energy production where it is used and connection to distribution lines with no need for transmission lines. Gallons Economics One of the biggest economic challenges relates to the fixed and semi-fixed soft costs and processes of development. They put a disproportionate burden on small projects. Site assessment costs, civil and geo-technical engineering costs, approval related costs, electrical engineering and connection assessment related costs. Either of these projects likely bear a disproportionate amount of soft costs relative to larger projects based on the fact that many of these tasks are performed exactly the same as they would be for larger projects. Therefore to make micro hydro possible something has to change, something has to be done differently from the current processes used for larger hydro project development in order to change the economics. Our focus therefore has been to attempt changes that reduce cost or waste in several ways; In order to reduce project design costs we have developed our own in house model working with several universities. The model inputs are head and flow duration data, and the model solves in about a week to find the optimal screw configuration and size that will maximize the rate of return on the project. The model considers approximately 20 million different possible screws, the cost of each, the revenues of each given the head and flow duration data and the rate of return of each. It also produces Solidworks drawings for the chosen screw. This not only eliminates most of the soft costs but it ensures that the last drop of rate of return is captured. To reduce the expensive site assessment costs necessary to determine head and flow duration data at many ungauged sites we are involved with two additional university research projects, both of these projects will increase the accuracy of our head and flow duration data, increase the speed with which a site assessment can be completed and eliminate much of the time and effort in collecting data in order to improve project economics. To understand the current nature of existing processes in an effort to eliminate waste and streamline processes we have vertically integrated into the design, manufacture and installations of systems as well as all approval processes with a focus on documenting and streamlining processes with a focus of making them standard and capable of installing a system a week. This approach has yielded large gains to date.

Applying the principle in reverse, the same equipment now offers a new method for generating power from water, providing a fish friendly and highly efficient alternative to a conventional turbine. Each Archimedean Screw hydropower system is manufactured to be site-specific, with a choice of designs depending on which is the most appropriate and cost-effective for each individual site. Why we use Archimedes Screw Generators The power that we can be derived from a hydro site is a function of the head, or height, of the vertical drop and the volume of water in the river. The bigger the head and the bigger the volume of water the more power can be generated. Hydro turbines at any specific site are designed to capture the greatest possible head and the greatest possible volume of water – Turbines are designed to turn when water goes through them and it is the turning motion which generates electricity. There are several types of hydro electric turbines. All the turbines have a gearing mechanism that drives the generator, but they differ in the manner by which the energy is extracted from the fall of water and converted into mechanical rotation. The mechanical rotation is then converted into electrical energy. We use Archimedes Screws because of these features; Fish Friendly – It, and the Waterwheel are the least harmful to fish of all the turbine types. The Archimedean Screw and the Waterwheel operate at normal atmospheric pressure while other turbines, for instance the Kaplan, operate by forcing water at high pressure into the system. In addition, the screw turns at a low velocity of approx 28 rpm. This, together with the operation of the turbine under normal atmospheric pressure, enables the safe passage of fish through the system. Although widely in use throughout Europe and the U. On the basis of previous studies carried out in Europe which supported the claims to fish-friendliness, the first Ontario Screw installation was installed in Literally thousands of fish passages have been monitored and recorded using underwater cameras at the intake, inside the chamber of the Screw itself and at the outflow to assess the effect of the Screw on salmonids including smolts and kelts , brown trout and eels. The trials looked at fish passage across a broad spectrum of sizes and turbine speeds, possibly the most impressive of which was the safe passage of a kelt measuring 98cm in length and weighing 7. The European studies conclude that the Archimedean Screw turbine is indeed fish- friendly with no adverse physical effect on fully grown fish or kelts; at most 1. In addition, behavioural and migrational patterns across the species have been shown to be entirely unaffected by the turbine. Fish safely entering the top of screw as the screw turns slowly. No Depleted Reach – this is measured as the distance between where the water leaves a river to enter a hydro system and where the water goes back into the river. Clearly by taking a given volume of water out of a river, there will be an impact upon the natural ecology of the river bed. A large depleted distance may therefore compromise the ecology of the river bed. The smaller the depleted distance, the less impact there will be upon the fish and wildlife ecology. The Archimedean Screw has the lowest depleted reach because it is placed alongside a river weir – the water enters the system from just above the weir and returns to the river just below the weir. Visibility – because the Archimedean Screw is visible, there is an educational benefit to this type of turbine. It can be readily viewed while in operation and the turning motion and the generation of electricity is immediately obvious.

Let us understand Archimedes' principle with the help of an activity.

He happened to be the son of an astronomer named Pheidias; and perhaps this fact played some role in the selection of his career. Like Einstein and Newton, he was so absorbed in mathematical and scientific inquiry that he would neglect his health and hygiene. We are also told that he deliberately inserted false theorems into his works especially in his work *The Sphere and the Cylinder*, partly to tease his friends, and partly also to identify thieves who would steal his ideas. Europe would not catch up to him until Newton himself definitively produced the mathematics of calculus, and that would be over years later. His treatises remain masterful examples of mathematical art; and we are left breathless in wonder at how one mind could have achieved so much in so little time. Of his published output, ten works have survived. The titles alone say it all: No account of Archimedes would be complete without relating the famous story of his discovery of the law of weight displacement in liquids. King Hieron of Syracuse, we are told, had hired an artisan to fashion a gold crown for him. To this end the king supplied him with some gold. The finished crown weighed the same as the allotment of gold provided, but the king had suspicions that the crown may have been alloyed with some cheaper base metal. Archimedes was brought in to consult on the problem. How could a way be found of learning the truth, without destroying the crown? The great scientist finally noticed one day, when bathing, that his body seemed to displace water in relation to the depth of his immersion. His body also seemed to weigh less the deeper it was pushed into water. From this, he was able to deduce that the weight of water displaced by a floating body is equal to the weight lost by the floating body. Whether this discovery actually prompted the scientist to run through the street shouting Eureka, Eureka, I will permit the skeptical reader, alert to the amusing apocrypha of history, to decide for himself. But how would this discovery help him in solving the problem of the crown? A base metal or silver would weigh much less than gold. Therefore a given weight of a base metal would have more volume than the same weight of gold. And it would also, therefore, displace more water. The only possible explanation could be that the crown had been alloyed with a cheaper base metal or silver. Archimedes was even able to calculate just how much gold had been stolen. The king, in other words, had been short-changed. Archimedes became famous for discovering how to measure specific gravity. The artisan was executed. He was also an eminently practical scientist, in an age when theoretical speculation too often took precedence over real-world application. He constructed a functional planetarium. He worked out the laws of pulleys and levers so completely that no improvement was made to his work until Like Leonardo da Vinci many centuries later, he was fascinated by weapons and machines of war. He did these tasks not out of a love for war, but simply out of his interest in solving problems of mechanics and physics. He also made practical advances in agriculture and irrigation that are used to this day: When the Romans undertook the siege of Syracuse as part of their expansion in the Italic peninsula, the old scientist then seventy-five was compelled to assist in the defense of the city. The historian Polybius I. Such a great and marvelous thing does the genius of one man show itself to be when properly applied. The Romans, strong both by land and sea, had every hope of capturing the town at once if one old man of Syracuse were removed; but as long as he was present, they did not venture to attack. But time did tell in this matter. After eight months of siege, the Roman commander, Marcellus, was able to take the starving citadel. He was a great general and a good man in his own right, and he gave strict instructions that Archimedes was not to be harmed. Yet such are the passions of war and conquest that they often overwhelm the intentions of decent men. The old scientist was found—by a Roman soldier unaware of his significance. The soldier ordered him out of his house, but Archimedes asked him to wait a moment until he could solve a problem he was working on. In the dispute that followed, he was slain. Marcellus ordered compensation for his relatives, and had a grave marker erected in his memory. He believed that the discovery of the formulas for the volumes of these things was his greatest achievement. With him, classical science went to the grave. Scientific inquiry would not achieve such heights for another years.

Archimedes was born in the city of Syracuse on the island of Sicily in BC. He was the son of an astronomer and mathematician named Phidias. Aside from that, very little is known about the early life of Archimedes or his family.

This activity provides students with an opportunity to use model-building as a way to help understand the forces and phenomena at work in the world around them. Both successful and unsuccessful models allow students to make inferences, refine hypotheses, and draw conclusions about the behavior of materials and structures. All of these are important aspects of the type of inquiry we call science. Lesson Background and Concepts for Teachers Information about Boat Designs Boat Hulls -- Form and Function With photographs from books and magazines, after students complete the Clay Boats activity, they can compare their own designs to boats commonly used for trade and recreation, both past and present. They can be guided through observations about the trade-offs between speed how fast the boat can go with a given power source , stability how likely the boat is to tip over under a given sideways force , draft how deeply the boat rides in the water , and cost how expensive a given design is to build. As students consider the different types of boats and their features, try to emphasize the relationships between the design, or form, of the boat, and its function. The more successful of the student-designed clay boats will probably resemble a flat-bottomed bowl. This design will hold many washers -- as long as the weight is carefully distributed in the boat. This is a feature of flat-bottomed boats: A distinct advantage of flat-bottomed boats is that they have a shallow draft, meaning their hulls do not extend very far down below the surface of the water compared to other hull shapes see Figure 1. Flat-bottomed boats are thus desirable for moving around in shallow water. Their simple shape also makes them the least expensive type of boat to build. Flat hulls are typically found in small utility boats such as Jon boats, and were commonly used in the last century as barges to transport goods on the quiet waters of canals in this country and in parts of Europe. In this case the flat hull is designed to rise up and ride on top of the water rather than cutting through the water, thereby encountering the reduced friction of moving through air instead of water see Figure 2. Although it takes a lot of engine power to get the hull up, at which point the boat is said to plane, it can then travel at very high rates of speed. Even when planing, the back, or stern, of the boat is still in the water. Tapered ends certainly let a boat move through the water more efficiently than a bowl-shape, since water can easily flow around the front bow of the boat if it is tapered. The rounded hull, however, presents a problem because such boats roll easily and take on water or capsize. Large sailboats, fishing trawlers, and cargo ships, which do have rounded hulls, generally also have keels. Because the keel extends down into the water, these boats cannot travel in shallow water the way boats with flat bottoms can. With their complicated hull shapes, these boats are also expensive to build. Multi-hulled boats, such as catamarans, trimarans, pontoon boats, and some house boats, are very stable due to their wide stance in the water. Each of the hulls can be flat, but usually they are either round or V-shaped. Multi-hulled boats are usually the most expensive to build. Superstructures and Center of Gravity The hull shape is the main determinant of how the boat interacts with the water, but real boats carry structures and cargo above their decks, too. Ask students how they think a tall superstructure would affect a ship when strong winds blow from the side. Also ask how a tall superstructure would affect a ship if it rolled to one side due to large waves. If there is time and student interest, you could provide materials such as Popsicle sticks and white glue, and challenge students to make the tallest floating superstructures they can for their boats. Students should be able to realize that it is necessary to keep the center of gravity as close to the midline of the ship as possible. Once the center of gravity is beyond the deck of the ship, it will tip over just as the towers tipped over once their centers of gravity got beyond their bases. Ask students where they think heavy cargo should be placed on a ship. Point out that ships carry ballast, or extra weight usually in the form of scrap metal , in their keels for the purpose of keeping the center of gravity low and along the midline of the ship. You can also ask students to speculate on the comparative keel depths of ships with lots of superstructure versus those with little superstructure. They are asked to repeat the procedure using a different amount of clay the second time in order to generalize the phenomenon. After completing the activity, students can look back at the water levels

they marked on their beakers to verify that the floating boat displaced more water than the sunken lump of clay did, a result that may have surprised them. Clay, therefore, can be a floater or a sinker, depending on its shape. It is denser than water, so ordinarily it sinks. But it can also be molded into a shape designed to displace a lot of water. In order to really understand what is going on with buoyancy, it is necessary to understand the idea of water pressure. Think of a large container of water. Water, because it is made of atoms and molecules, has mass, and the mass of the water near the surface pushes down on the water near the bottom. In other words, the water below is under pressure due to the mass of the water above. Actually, the water at the surface is also under pressure, due to the mass of the atmosphere pressing upon it, but this pressure is much lower than the pressure at the bottom of the container. One thing that is interesting about fluid pressure is that it is a type of force that acts in all directions at once. In contrast to gravity, which only acts downwards, water pressure pushes against any object it contacts, regardless of the orientation or location of the object within the fluid. What this means is that if an object such as a block of wood is placed in the water, gravity acts to pull downwards on the block tending to make it sink, but at the same time, water pressure acts upwards against the block. The water pressure provides the buoyant force. To understand this concept, it may help students to think about three blocks, each in the shape of a cube that is one foot on a side. One block is made of solid wood, and a cubic foot of wood weighs about fifty pounds. Another block is also made of wood, but it has been hollowed out in the middle, so it weighs only 10 pounds. If you put all three blocks in a pool of water, they would all float, since they are all less dense than the water. However, the blocks would not float in quite the same way. The solid block would ride low in the water, as shown in the figure below. The water pressure is slight up at the surface of the water, but since very little pressure is needed, the foam block does not sink very deeply into the water. The hollow wooden block, however, has to ride lower in the water in order to encounter enough water pressure to keep it afloat. The solid wood block, meanwhile, rides quite low in the water compared to the other two blocks. It has to extend even more deeply into the water than the hollow block, down to where the water pressure is high enough to counteract its greater mass. But why does modeling clay, which is denser than water, float when it is given a bowl-like or boat-like shape? Or how do ships, for which steel is the main structural component, manage to float? The reason is that their shapes provide a large area for the water pressure to act against. The total buoyant force on an object equals the water pressure at its floating depth times the area of the object in contact with the water. This means that the more area you can give a material, the higher it can ride on the water, where the water pressure is much less. If the solid cube of wood in Figure 1 were stretched out into a plank that was eight feet long, nine inches wide, and 2 inches high, it would have the same weight and volume as the cube, but it would float nicely right at the surface of the water. This is because the plank would have six square feet of area for the water pressure to push against instead of only one square foot the block has. The ability to float in a liquid or rise in a gas. The mass per unit volume of a substance at a given pressure and temperature. Associated Activities Clay Boats - Using only a limited amount of modeling clay, students are challenged to design an object that not only floats, but will carry the largest load possible. Watch this activity on YouTube Buoyant Boats - Students measure the amount of water displaced by a lump of modeling clay when it is shaped so as to sink in water, and then compare that amount to the amount of water the same piece of clay displaces when it is shaped so as to float in water. Assessment Provide students with a copy of the diagram shown in Figure 1, and ask them to write a paragraph that explains why the three objects do not float in water in quite the same ways. Ask students to write a paragraph that explains why a lump of clay will sink in water, but the same volume of clay, when shaped like a bowl, will float in water. Similarly, you can ask students why a bar of steel will sink in water, but ships made of steel do not. Give them the option of including diagrams with their explanations. Ask students to predict the weight of water that would be displaced by an empty canoe weighing pounds. Assume the canoe is afloat. Also, ask if the amount of water displaced by the same canoe would increase or decrease if the canoe tipped over, filled with water, and sank. Lesson Extension Activities Ask students if they themselves are floaters or sinkers. Ordinarily, humans float, but barely. Our bodies contain mostly water, but we do have minerals in our bones, which are denser than water, and air in our lungs even when we exhale fully, there is still some residual air inside, which is much less dense than water. No matter how thin we are, we all still have some amount of

fat in our bodies, which is less dense than water. How easily we float depends largely on how much body fat we have. We can also float better if we fill our lungs and hold our breath. You can also ask your students if it is easier to float in a lake fresh water or in the ocean salt water. They should remember, if they have had any experience, that it is easier to float in the latter. Ask them why this is true, and give them some time to try to work out this puzzle. You may need to remind them what density really means; it is the amount of "stuff" mass packed into a given space, and liquids and gases have densities, too. Ocean water has salt, a type of "stuff", dissolved in the water. With more stuff in it, ocean water is denser than fresh water. Put another way, a gallon of salt water weighs more than a gallon of fresh water. Because of this weight difference, when an object floats in salt water, a smaller volume of water needs to be displaced than would be needed if the water contained no salt. Because less salt water needs to be displaced, an object floats higher in the salt water. Students can design some simple experiments to compare the volumes of displaced liquid when clay boats float in fresh versus salt water. Sea water contains about 3. You can also ask your students what would happen when a fully loaded cargo ship travels from the ocean up into a large river such as the Amazon. They can simulate this experience by making a clay boat and filling it with as many metal washers as it will hold without sinking in salt water. Then they can carefully move the boat and its load into a container of fresh water, where it will most likely sink. If students have ever swum in a deep lake in the summer, they may have discovered that the surface water was warm, but if they dove down a few feet, the water became cooler. The colder, denser water remains at the bottom, unless something physically causes the warm and cool water to mix. In a relatively shallow swimming pool, the action of swimmers is enough to keep the water mixed. Students can use their clay boats to try to compare the densities of very hot and very cold water.

Chapter 9 : Archimedes' Principle

Floating between two liquids. (e.g. coins, shot) until the floating in water (alone) is ensured. It is in a stable vertical position, the Archimedes' up.

Archimedes Quotes The work done by Archimedes ca. He is considered one of the greatest mathematicians of all time. Archimedes probably was born in the seaport city of Syracuse, a Greek colony on the island of Sicily. He was the son of an astronomer, Phidias, and may have been related to Hieron, King of Syracuse, and his son Gelon. Archimedes studied in Alexandria at the school established by Euclid and then settled in his native city. To the Greeks of this time, mathematics was considered one of the fine arts—something without practical application but pleasing to the intellect and to be enjoyed by those with the requisite talent and leisure. Archimedes did not record the many mechanical inventions he made at the request of King Hieron or simply for his own amusement, presumably because he considered them of little importance compared with his purely mathematical work. These inventions did, however, make him famous during his life. Fact and Fancy The many stories that are told of Archimedes are the prototype of the absentminded-professor stories. A famous one tells how Archimedes uncovered a fraud attempted on Hieron. The King ordered a golden crown and gave the goldsmith the exact amount of gold needed. The goldsmith delivered a crown of the required weight, but Hieron suspected that some silver had been used instead of gold. He asked Archimedes to consider the matter. Once Archimedes was pondering it while he was getting into a bathtub full of water. He noticed that the amount of water overflowing the tub was proportional to the amount of his body that was being immersed. There are several ways Archimedes may have determined the proportion of silver in the crown. Using this method, he would have first taken two equal weights of gold and silver and compared their weights when immersed in water. Next he would have compared the weight of the crown and an equal weight of pure silver in water in the same way. The difference between these two comparisons would indicate that the crown was not pure gold. On another occasion Archimedes told Hieron that with a given force he could move any given weight. Archimedes had investigated properties of the lever and pulley, and it is on the basis of these that he is said to have asserted, "Give me a place to stand and I can move the earth. In the harbor was a new ship which the combined strength of all the Syracusans could not launch. Archimedes used a mechanical device that enabled him, standing some distance away, to move the ship. The device may have been a simple compound pulley or a machine in which a cogwheel with oblique teeth moves on a cylindrical helix turned by a handle. Hieron saw that Archimedes had a most inventive mind in such practical matters as constructing mechanical aids. At this time one use for such inventions was in the military field. Hieron persuaded Archimedes to construct machines for possible use in warfare, both defensive and offensive. A Time of War Plutarch in his biography of the Roman general Marcellus describes the following incident. After the death of Hieron, Marcellus attacked Syracuse by land and sea. But Archimedes began to work his engines and hurled against the land forces all sorts of missiles and huge masses of stones, which came down with incredible noise and speed; nothing at all could ward off their weight, but they knocked down in heaps those who stood in the way and threw the ranks into disorder. Furthermore, beams were suddenly thrown over the ships from the walls, and some of the ships were sent to the bottom by means of weights fixed to the beams and plunging down from above; others were drawn up by iron claws, or crane-like beaks, attached to the prow and were plunged down on their sterns, or were twisted round and turned about by means of ropes within the city, and dashed against the cliffs. Marcellus, according to Plutarch, gave up trying to take the city by force and relied on a siege. The city surrendered after 8 months. Marcellus gave orders that the Syracusan citizens were not to be killed, taken as slaves, or mistreated. But some Roman soldier did kill Archimedes. There are different accounts of his death. One version is that Archimedes, now 75 years old, was alone and so absorbed in examining a diagram that he was unaware of the capture of the city. A soldier ordered him to go to Marcellus, but Archimedes would not leave until he had worked out his problem to the end. The soldier was so enraged, he killed Archimedes. Another version is that Archimedes was bringing Marcellus a box of his mathematical instruments, such as sundials, spheres, and angles adjusted to the apparent size of the sun, when he was killed

by soldiers who thought he was carrying valuables in the box. The Roman statesman and writer Cicero tells of finding this tomb much later in a state of neglect. Other Inventions Perhaps while in Egypt, Archimedes invented the water screw, a machine for raising water to irrigate fields. Another invention was a miniature planetarium, a sphere whose motion imitated that of the earth, sun, moon, and the five other planets then known Saturn, Jupiter, Mars, Venus, and Mercury ; the model may have been kept in motion by a flow of water. Cicero tells of seeing it over a century later and claimed that it actually represented the periods of the moon and the apparent motion of the sun with such accuracy that it would, over a short period, show the eclipses of the sun and moon. Archimedes is said to have made observations of the solstices to determine the length of the year and to have discovered the distances of the planets. But the problems Archimedes set himself and his solutions were on another level from any that preceded him. In Book XII the method of exhaustion, discovered by Eudoxus, is used to prove theorems on areas of circles and volumes of spheres, pyramids, and cones. Two of the theorems are mentioned by Archimedes in the preface to *On the Sphere and Cylinder*. After stating the result concerning the ratio of the volumes of a cylinder and an inscribed sphere, he says that this result can be put side by side with his previous investigations and with those theorems of Eudoxus on solids, namely: There was no direct computation of areas and volumes enclosed by various curved lines and surfaces, but rather a comparison of these with each other or with the areas and volumes enclosed by rectilinear figures such as rectangles and prisms. The reason for this is that the area, for a simple example, of a circle with radius of length one cannot be expressed exactly by any fraction or integer. It is possible, however, to say as is done in Proposition 2 of Book XII of the *Elements* that the ratio of the area of one circle to another is exactly equal to the ratio of the squares of their diameters, or, in a more concise form closer to the Greek, circles are to one another as the squares of the diameters. The proof of this theorem relies on theoretically being able to "exhaust" the circle by inscribing in it successively polygons whose sides increase in number and hence which fit closer to the circle. Thus the curved line, the circle, can be closely approximated by a rectilinear figure, a polygon. Recognizing this, it would be easy to conclude that the circle itself is a polygon with "infinitely" many "infinitesimal" sides. Archimedes, aware of the logical problems involved in making such a facile statement, avoids it and proceeds in his proofs in an invulnerable manner. It should nevertheless be remembered that the theorems which make the work almost trivial to any modern mathematician were obtained only in the 17th, 18th, and 19th centuries, about years after Archimedes. Large numbers seem to have some fascination of their own. A common Greek proverb was to the effect that the quantity of sand eludes number, that is, is infinite. To the Greeks this might seem especially true since their numeral system did not include a zero. Numbers were represented by letters of the alphabet, and for large numbers this notation becomes clumsy. In *The Sand Reckoner* Archimedes refutes the idea expressed by the proverb by inventing a notation which enables him to calculate in a reasonably concise way the number of grains of sand required to fill the "universe. After saying this he also points out an alternative view of the universe that had been expressed by a contemporary astronomer, Aristarchus of Samos, namely, that the sun is fixed, the earth revolves about the sun, and the stars are fixed a long distance beyond the earth. Astronomical data, together with the assumption that there are no more than 10¹⁰ grains of sand in a volume the size of a poppyseed, are the basis of calculations leading up to the conclusion that the number of grains of sand which could be contained in a sphere the size of the universe is less than 10²¹, in modern notation. The first is concerned with volumes of segments of such figures as the hyperboloid of revolution. The third is on finding areas of segments of the parabola. From such simple postulates as "Equal weights at equal distances balance," positions of centers of gravity are determined for parabolic segments. As is true of all other mathematicians of antiquity, Archimedes usually wrote in a way which left no indication of how he arrived at the theorems; all the reader sees is a theorem followed by a proof. But in a hitherto-lost treatise by Archimedes, *The Method*, was found. In it Archimedes explains a certain method by which it is possible to get a start in investigating some of the problems in mathematics by means of mechanics. The *Method* utilizes theorems from his mechanical treatise *On the Equilibrium of Planes* and provides an excellent example of the interplay between pure and applied mathematics. For biographical information see E. Dijksterhuis, *Archimedes* ; trans. Works on mathematics for the general reader are Thomas L. See also Robert S. Brumbaugh, *Ancient Greek Gadgets and Machines*

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